HÖLDER REGULARITY FOR A CLASSICAL PROBLEM OF THE CALCULUS OF VARIATIONS

The talk is devoted to a recent result obtained jointly with Giulia Treu. Let \( \Omega \subset \mathbb{R}^n \) be bounded, open and convex, \( F : \mathbb{R}^n \to \mathbb{R} \). Consider the problem of minimizing

\[
I(u) = \int_{\Omega} F(\nabla u(x)) \, dx
\]

among the functions of \( W^{1,1}_0(\Omega) \) that have a prescribed trace \( \phi \) on \( \partial \Omega \). Existence of a solution follows classically from Tonelli’s assumptions on \( F \), namely that \( F \) is convex and superlinear. The classical regularity theory [3] then implies that the minimizers of \( I \) are Hölder continuous if \( F \) has a polynomial growth, i.e. \( a|\xi|^p \leq F(\xi) \leq b|\xi|^p \) for some \( p > 1 \), with \( a, b > 0 \). We are concerned with the case where the boundary datum \( \phi \) is Lipschitz. Unless \( \phi \) does satisfy some more restrictive conditions (e.g. barriers, like the Bounded Slope Condition) one cannot expect that the minimizers of \( I \) are Lipschitz. We show however that the \( p \)-coercivity from below is sufficient to ensure that the minimizers of \( I \) are globally Hölder continuous of a specific order, no matter what is the growth of \( F \) from above ([5], announced in [1]). When the domain is a polyhedron the result was established, with a smaller Hölder rank and only for strictly convex lagrangians, by Clarke [2]: the main tools of [2] are a new dilation technique and a comparison principle for minimizers that we established in recent years. Their improvement to face the situation when \( F \) is not strictly convex [4], the idea of comparing solutions on different domains and a Rado–Haar type theorem for Sobolev functions are the ingredients of our proof.

References


