KRULL-SCHMIDT FINITELY ACCESSIBLE CATEGORIES

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Abstract. An additive category is called finitely accessible if it has direct limits, the class of finitely presented objects is skeletally small, and every object is a direct limit of finitely presented objects. Any finitely accessible additive category $\mathcal{C}$ may be embedded as a full subcategory of the category $\text{Mod}(A)$ of unitary right modules over the functor ring $A$ of $\mathcal{C}$ such that the pure exact sequences in $\mathcal{C}$ are those which become exact sequences in $\text{Mod}(A)$ through the embedding. The induced equivalence between $\mathcal{C}$ and the full subcategory of the category $\text{Mod}(A)$ consisting of flat right $A$-modules offers the main technique for translating properties of modules over the functor ring $A$ to properties of the finitely accessible category $\mathcal{C}$. Let $\mathcal{C}$ be a finitely accessible additive category with products, and let $(U_i)_{i \in I}$ be a family of representative classes of finitely presented objects in $\mathcal{C}$ such that each object $U_i$ is pure-injective. Using functor ring techniques, we show that $\mathcal{C}$ is a Krull-Schmidt category if and only if every pure epimorphic image of the objects $U_i$ is pure-injective. We discuss connections with the classical Osofsky theorem (which characterizes semisimple rings as those rings for which every cyclic module is injective) and with a Grothendieck categorical version of the Osofsky-Smith theorem for modules.