Closed-form Expansions for Option Prices in
Time-change Based Models

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Abstract
Within the area of pricing and hedging equity derivatives, models combining jumps with stochastic volatility have attracted considerable attention due to their capability of capturing the informational content of option prices across strikes and maturities at one point in time. Log-prices are typically constructed by time-changing a Lévy process with a random clock capturing the desired features of stochastic volatility and we refer to this modeling approach as time-changed Lévy models Carr & Wu (2004). If the rate of time-change is selected as an affine process, the pair comprising log-price and volatility retains the affine structure allowing for closed-form option pricing via Fourier transform methods. Although fast and accurate, these numerical techniques act somewhat as black-boxes and fail to provide a direct link between the structural properties of the models and their effect on the characteristics of the generated volatility surface. In this work we derive a simple expansion of option prices in terms of Black-Scholes prices and Greeks which highlights and quantifies the impact of jumps and stochastic volatility in a transparent manner by means of convexity and cross-convexity adjustments. This approach is inspired by the work of Drimus (2011) where only the Heston (1993) model is considered. Here, we extend the approach and apply it to the much wider class of option pricing models based on time-change where the subordinant Lévy process itself is obtained as a time-changed Brownian motion. This framework encompasses a vast variety of models which includes e.g. the NIGSV, VGSV, and CGMYSV models presented by Carr et al. (2003) and the non-Gaussian Ornstein-Uhlenbeck models considered in Bärdndorff-Nielsen & Shephard (2001). Furthermore, the approximation allows for faster and more efficient option pricing in e.g. affine models where the characteristic function is not explicitly available but only computable up to the solution of a Riccati ODE. Finally, full computations for the second and third order approximations are provided and examined for concrete specifications.

Keywords: Option pricing, stochastic volatility, Lévy processes, time-change, price expansions, convexity adjustments.