**Construction of random time for default models**

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ABSTRACT. In many models in Finance, default times are constructed starting from a given intensity through a Cox model, i.e., given a positive $\mathbb{F}$-adapted process $\lambda$ (the intensity rate), one defines a random time $\tau$ as $\tau = \inf\{t : \int_0^t \lambda_s ds > \Theta\}$ where $\Theta$ is a random variable with exponential law, independent of the reference filtration $\mathbb{F}$. This process enjoys the property that $M_t = \mathbb{1}_{\tau \leq t} - \int_0^{t \land \tau} \lambda_s ds$ is a martingale (in other words, $\Lambda$ is the intensity of $\tau$, where $\Lambda_t = \int_0^t \lambda_s ds$). Furthermore, any $\mathbb{F}$ martingale is a $\mathbb{G}$ martingale (immersion property) where $\mathbb{G}$ is the filtration $\mathbb{F}$, progressively enlarged with the random time $\tau$, and $\mathbb{P}(\tau > t | \mathbb{F}_t) = e^{-\Lambda_t}$.

In full generality, it is well known that, for a random time $\tau$, the conditional survival probability $G$ is a super-martingale, with a multiplicative decomposition $G_t = \mathbb{P}(\tau > t | \mathbb{F}_t) = N_t e^{-A_t}$ where $N$ is a $\mathbb{F}$-local martingale and $A$ an increasing $\mathbb{F}$-predictable process. Assuming that $A$ is continuous, it follows that $\mathbb{1}_{\tau \leq t} - A_{t \land \tau}$ is a martingale.

In this paper, we prove that, if an $\mathbb{F}$ supermartingale $Z$ valued in $[0, 1]$ admits a multiplicative decomposition $Z_t = N_t e^{-A_t}$ where $A$ is a continuous increasing process $A$, one can construct (infinitely many) random times $\tau$ such that $\mathbb{P}(\tau > t | \mathbb{F}_t) = N_t e^{-A_t}$. All these random times admit the same intensity process $\Lambda_t = A_t$. 