Title: Non-Lipschitz points and the $SBV$ regularity of the minimum time function

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Abstract: We study the Hausdorff dimension of the set of non-Lipschitz points $S$ of the minimum time function $T$ under controllability conditions which imply the continuity of $T$. We consider first the case of normal linear control systems with constant coefficients in $\mathbb{R}^N$. We characterize $S$ as the set of points which can be reached from the origin by an optimal trajectory (of the reversed dynamics) with vanishing minimized Hamiltonian. Linearity permits an explicit representation of $S$. Furthermore, we show that $S$ is $H^{N-1}$-rectifiable and has positive $H^{N-1}$-measure.

Second, we consider a class of control-affine planar nonlinear systems satisfying a second order controllability condition: we characterize the set $S$ in a neighborhood of the origin in a similar way and prove its $H^1$-rectifiability and that $H^1(S) > 0$.

In both (linear and nonlinear) cases, $T$ is known to have epigraph with positive reach, hence to be a locally $BV$ function. Since the Cantor part of $DT$ must be concentrated in $S$, our analysis yields that $T$ is $SBV$, i.e., the Cantor part of $DT$ vanishes.

This talk is based on a joint work with Giovanni Colombo and Khai T. Nguyen.