Abstracts

► Francesco Baldassarri (Università di Padova)
Title: Non-archimedean analysis of irregular singularities

Abstract: Let $X$ be a smooth algebraic surface over the complex field $\mathbb{C}$, and $Z$ be a divisor with normal crossings in $X$. Sabbah’s conjecture on the existence of a good formal structure for an integrable connection on $X$ with poles along $Z$, has been proved independently by Kiran Kedlaya and Takuro Mochizuki.

I will reconsider here Kedlaya’s strategy. I will first describe the structure of the space of valuations centered on the formal tube of $Z$ in $X$. I will then show how to reduce the study of variation of irregularity at a germ of transversal curve, to the study of the $t$-adic radii of convergence of solutions of a connection induced on a $\mathbb{C}((t))$-analytic curve, the “generic deformation of $Z$ in $X$”. Sabbah’s conjecture is reduced to the existence of a $\mathbb{C}[[t]]$-semistable model (after ramification of $t$ and blow-up) of this curve, which “controls” the behaviour of the non-archimedean radii of convergence. This fact is now understood, although still unpublished (work of Kedlaya, Pulita, Poineau, and the speaker).

We finally examine the notion of cleanness of $Z$ w.r.t. the given connection, and obtain, under this assumption, an index formula for the E-P characteristic of the De Rham complex, generalizing a rank 1 formula of Kato.

► Philip Boalch (ENS, Paris)
Title: Fission and wild character varieties

Abstract: As is well known Hitchin showed that the complex character varieties of Riemann surfaces (spaces of representations of the fundamental group of a Riemann surface) are hyperkahler manifolds and so admit a family of complex structures. In particular they have a special complex structure, the Higgs bundle complex structure, in which they are fibred by abelian varieties (the Hitchin fibration). On the other hand Deligne’s 1970 work on Hilbert’s 21st problem shows that complex character varieties of punctured smooth complex algebraic curves parametrise linear algebraic connections with regular singularities at the punctures. Hitchin’s hyperkahler story was extended to this case by Simpson, Nakajima and others in the 1990’s. In this talk I will describe the “wild character varieties” (which appear when we omit Deligne’s regularity assumption) and sketch an algebraic way to construct their natural complex symplectic structures. This involves a new operation called “fission” (which is different to fusion).

► Bruno Chiarellotto (Università di Padova)
Title: Arithmetic monodromy

Abstract: The classical theory for a topological semistable degeneration together with the associated behaviors of nearby/vanishing cycles and monodromy can be translated in several ways also in arithmetic algebraic geometry framework. In this talk we will focus on some recent results on the case of a scheme defined over a DVR having semistable reduction and on the case of a family of varieties in characteristic $p$ having a semistable degeneration. Of course, in this arithmetic algebraic setting, the way to the expected results will be paved by the classical
case, but non-classical ones will be obtained, too. The results have been presented in a series of articles written together with R. Coleman, V. Di Proietto, A. Iovita, N. Tsuzuki.

► **Alexander Getmanenko** (*Université de Paris 6*)

**Title:** Microlocal properties of sheaves and complex WKB  
**Abstract:** In a joint work with Tamarkin we apply Kashiwara-Schapira style microlocal theory of sheaves in order to justify an analytic continuation of solutions of the Laplace transformed Schrödinger equation. This work is motivated by foundational questions of complex WKB and resurgence theory of Ecalle, Voros, et al.

► **Michel Granger** (*Université d’Angers*)

**Title:** Gevrey expansion of hypergeometric integrals  
**Abstract:** We study the integral representations of the Gevrey series solutions of irregular hypergeometric systems. In this talk we shall consider more specifically the case of a system associated with a one row matrix, i.e., with a monomial curve. We prove that any Gevrey series solution along the singular support of the system is the asymptotic expansion of a holomorphic solution of a carefully chosen integral representation. This result holds even for the special values of the parameter. We shall end by giving a hint of what happens in higher dimension.

► **Julien Grivaux** (*Université de Provence*)

**Title:** New progress on the Grothendieck-Riemann-Roch theorem in Deligne cohomology  
**Abstract:** In this talk, we will discuss some known and some open cases of the Grothendieck-Riemann-Roch theorem for abstract complex manifolds. In Deligne cohomology, the GRR theorem predicts an explicit formula for the Knudsen-Mumford determinant of a holomorphic vector bundle attached to a submersion. We will explain how to refine results of Bismut-Gillet-Soulé and Bismut in order to prove a variational formula for the Knudsen-Mumford determinant.

► **Stéphane Guillermou** (*Université de Grenoble I*)

**Title:** Quantization of conic Lagrangian submanifolds of cotangent bundles  
**Abstract:** Several recent works make use of the microlocal theory of sheaves of M. Kashiwara and P. Schapira to obtain results in symplectic geometry. The link between sheaves on a manifold $M$ and the symplectic geometry of the cotangent bundle of $M$ is given by the microsupport of a sheaf, which is a conic co-isotropic subset of the cotangent bundle. In the above mentioned works properties of a given Lagrangian submanifold $\Lambda$ are deduced from the existence of a sheaf with microsupport $\Lambda$, which we call a quantization of $\Lambda$.

In this talk I will give sufficient conditions on a Lagrangian submanifold which imply the existence of a quantization. We will see how to use the quantization to recover results of Fukaya-Seidel-Smith and Abouzaid on the topology of a compact exact Lagrangian submanifold of a cotangent bundle.

► **Marco Hien** (*Universität Augsburg*)

**Title:** Stokes structures and the Fourier transform  
**Abstract:** There have been several attempts to understand the Stokes structure of the Fourier transform of a differential module on the affine line. I want to propose a topological way to attack these questions and report on recent results. This is a joint project with Claude Sabbah.

► **Naofumi Honda** (*Hokkaido University*)

**Title:** Multi-specialization and multi-microlocalization  
**Abstract:** The multi-microlocalization functor is a natural extension of the microlocalization functor along a single closed submanifold to that along several closed ones. This functor,
and the corresponding multi-specialization functor, gives important objects in analysis such as the sheaves of bi-microfunctions, those of strongly asymptotically developable holomorphic functions, and so on.

We will explain, in this talk, these new functors and give some applications. This is a joint work with L. Prelli and S. Yamazaki.

**Yves Laurent** *(Université de Grenoble I)*

**Title:** $L^2$ and singular solutions for holonomic $\mathcal{D}$-modules

**Abstract:** Three questions about holonomic $\mathcal{D}$-modules are closely related, namely the integrability of solutions, the existence of distribution solutions and the existence of submodules both supported by a subvariety. The study of these questions is quite different from other questions on growth of solutions because they are not stable by short exact sequences of $\mathcal{D}$-modules. In a previous work, we gave a partial answer to solve a classical Lie algebra question. Here we want to investigate more precisely these questions by means of resolution of singularities.

**Luca Migliorini** *(Università di Bologna)*

**Title:** Support theorem for relative Hilbert schemes of families of planar curves

**Abstract:** I will discuss joint work with V. Shende (MIT) proving an analogue of Ngo’s support theorem for relative Hilbert schemes of families of integral planar curves. As a corollary we have a “Macdonald formula” relating the cohomology of the Hilbert scheme of an integral, planar, singular curve to the cohomology of its compactified jacobian. If time permits I will discuss application to the theory of BPS states and also what happens in the case of reducible curves.

**Takuro Mochizuki** *(RIMS, Kyoto University)*

**Title:** Harmonic bundles and Toda lattices with opposite sign

**Abstract:** First, we describe a classification of the real valued solutions of a Toda equation in terms of their parabolic weights, from the viewpoint of the Kobayashi-Hitchin correspondence. Then, we explain when the associated integrable variation of twistor structure has an integral structure. It follows from two results. One is the explicit computation of the Stokes factors of a certain meromorphic flat bundle. The other is an explicit description of the associated meromorphic flat bundle. We use the opposite filtration of the limit mixed twistor structure with an induced torus action.

**Luis Narváez Macarro** *(Universidad de Sevilla)*

**Title:** On the symmetry of the roots of Bernstein-Sato polynomials

**Abstract:** It is well known that the roots $\neq -1$ of the Bernstein-Sato polynomial of a quasi-homogeneous isolated hypersurface singularity in ambient dimension $d$ are symmetric with respect to $-d/2$. Recently, Granger and Schulze discovered that the Bernstein-Sato polynomial of any reductive prehomogeneous determinant or of any special linear free divisor satisfy the equality $b(-s - 2) = \pm b(s)$, and so its roots are symmetric with respect to $-1$. Motivated by these results, the author has proven that the equality $b(-s - 2) = \pm b(s)$ also holds for the Bernstein-Sato polynomial of any free divisor $h = 0$ for which the $\mathcal{D}[s]$-module $\mathcal{D}[s]h^s$ has a “Spencer logarithmic” resolution. This hypothesis is satisfied by any free divisor of “linear Jacobian type”, and in particular by any locally quasi-homogeneous free divisor (e.g. a free hyperplane arrangement) (see [http://arxiv.org/abs/1201.3594](http://arxiv.org/abs/1201.3594)).

Since the codimension of the singular locus of any (singular) free divisor is 2, we can think that the above examples are extremal cases of other intermediate symmetries.

In this lecture I will give some ideas of the proof of the above symmetry in the case of free divisors, based on the behaviour of $\mathcal{D}[s]h^s$ under duality, and I will show how to construct
examples of hypersurfaces for which the roots \( \neq -1 \) of their Bernstein-Sato polynomial are symmetric with respect to \(-e/2\), where \( e \) is the codimension of the singular locus. These examples support a conjecture that will be presented in the talk.

■ Claude Sabbah  (École Polytechnique)
Title: On the irregular Hodge filtration
Abstract: Given a regular function \( f \) on a smooth complex quasi-projective variety, we generalize the construction of Deligne (1984) of the “irregular Hodge filtration” on the hypercohomology of the de Rham complex with twisted differential \( d + df \) and we show a \( E_1 \)-degeneracy property. One proof uses the theory of twistor \( D \)-modules. We will also report on another approach by M. Kontsevich, and the corresponding proofs by Kontsevich (by char. \( p \) methods) and Morihiko Saito (by Steenbrink-Hodge theoretical methods). This is a joint work with Hélène Esnault and Jeng-Daw Yu.

■ Christian Schnell  (Stony Brook University)
Title: Holonomic \( D \)-modules on abelian varieties
Abstract: The Fourier-Mukai transform is an equivalence of categories between (the bounded derived categories of) coherent \( D \)-modules on an abelian variety and coherent sheaves on the moduli space of line bundles with connection. I will describe some results about Fourier-Mukai transforms of holonomic \( D \)-modules, and outline a conjectural structure theorem, which relates them to a (yet to be defined) category of hyperkahler perverse sheaves.

■ Christian Sevenheck  (Mannheim University)
Title: Toric geometry, hypergeometric \( D \)-modules and mirror symmetry
Abstract: I will explain how mirror symmetry for toric manifolds with numerically effective anticanonical bundle (e.g. Fano varieties) can be understood using filtered \( D \)-modules. This yields a construction of non-commutative Hodge structures associated to the mirror of such a variety. For a nef complete intersection in a toric variety, I will describe a non-affine Landau-Ginzburg model associated to it, and show that the mirror is given by the intersection cohomology of the fibres of this map.

■ Dmitri Tamarkin  (Northwestern University)
Title: Microlocal category for a closed symplectic manifold
Abstract: I will discuss how, using Microlocal Analysis of constructible sheaves by Kashiwara-Schapira, one can associate to a dg-category a category whose symplectic form has integer periods. The resulting category is defined over Novikov’s ring and resembles very much the celebrated Fukaya’s category.

■ Motoo Uchida  (Osaka University)
Title: TBA

■ Kari Vilonen  (Northwestern University)
Title: The microlocal codimension three conjecture
Abstract: I will explain the proof of the microlocal codimension three conjecture. The conjecture states that any (regular) holonomic \( E \)-module uniquely extends across a codimension three locus on its support. This is joint work with M. Kashiwara.

■ Giuseppe Zampieri  (Università di Padova)
Title: Geometric and analytic methods for propagation of regularity of CR functions
Abstract: We describe the objects of the boundary of a domain in the complex space which are propagators of holomorphic extendibility for CR functions. We prove that degenerate analytic discs, and even real curves, are propagators if their singularity has a good balance with the “type” of the boundary.

Short Communications

► Francisco Javier Calderón Moreno (Universidad de Sevilla)
Title: Integrable logarithmic connections over the cusp
Abstract: In this talk, I want to expose a joint work with Luis Narváez.

Let $X$ be a $n$-dimensional complex analytic manifold, $D \subset X$ a divisor, $\mathcal{E}$ an integrable logarithmic connection with respect to $D$ and $\mathcal{L}$ the local system of the horizontal sections of $\mathcal{E}$ on $U = X \setminus D$.

We will show the progress we have achieved to obtain a classification of the integrable logarithmic connections $\mathcal{E}$ when $D$ is the cusp. This classification can be used to study properties of $\mathcal{E}$, as the irreducibility of $D \otimes \mathcal{E}$ as $D$-module, and so we can study the properties of the local system $\mathcal{L}$.

► María Cruz Fernández Fernández (Universidad de Sevilla)
Title: Asymptotic behavior of hypergeometric functions along toric curves
Abstract: This is joint work with F. Castro, N. Takayama and T. Koike [1]. A modified A-hypergeometric system [6] is a system of linear partial differential equations in the complex variables $x_1, \ldots, x_n, t$ for the function $f(t^{w_1} x_1, \ldots, t^{w_n} x_n)$ where $f(y_1, \ldots, y_n)$ is a holomorphic solution of an A-hypergeometric system (or GKZ-system after [4]) and $w \in \mathbb{Z}^n$ is called the weight vector. We study the irregularity of modified systems by adapting to this case the notion of umbrella introduced in [5]. Especially, we study slopes and Gevrey series solutions. We develop some applications of this study. First, we give Laplace integral representations of divergent series solutions of the modified system under some conditions on the weight vector and then we give an analytic meaning to the Gevrey series solutions of the original A-hypergeometric system along coordinate varieties as constructed in [2] and [3]. More precisely, we obtain that they are asymptotic expansions of hypergeometric functions along toric curves.

References


► Jorge Mozo Fernández (Universidad de Valladolid)
Title: Asymptotics with respect to an analytic germ, and Ramis-Sibuya Theorem
Abstract: The notion of monomial asymptotics, developed by M. Canalis-Durand, J. Mozo and R. Schäfke in order to treat asymptotics of solutions of singularly perturbed differential equations, is extended in this work to a more general notion of asymptotics with respect to a germ of analytic function, in any number of variables. One of the main tools used is monomialization theorem, which is a consequence of the reduction of singularities for analytic germs of hypersurfaces. This result is also used in order to extend Ramis-Sibuya Theorems to this context. It is a joint work with R. Schäfke (Strasbourg University).

Mika Tanda (Kinki University)

Title: Borel sums of Voros coefficients for hypergeometric differential equations with a large parameter

Abstract: We consider classical hypergeometric differential equations from the viewpoint of the exact WKB analysis. One of main objects in the exact WKB analysis is the notion of WKB solutions which are formal solutions of the equations coming from microdifferential operator-valued solutions of the associated Riccati equations. Voros coefficients are formal series in the negative powers of the large parameter which describe the discrepancy between two WKB solutions with different normalization of integration and they play a role in the analysis of Stokes phenomena with respect to parameters in the equations. They are firstly introduced by Voros for the quartic oscillators. We define Voros coefficients for hypergeometric differential equations with a large parameter and give explicit forms of them. We show that the Borel sums of them can be computed explicitly.

Jean-Baptiste Teyssier (École Polytechnique)

Title: A differential module analogue of a construction of Abbes and Saito

Abstract: By using a blow-up construction, the nearby-cycle functor and ℓ-adic Fourier transform, Abbes and Saito are able to define a geometric measure of wild ramification of ℓ-adic sheaves on the generic point of any complete discrete valuation ring of equal characteristic $p \neq \ell$ with perfect residue field. After a brief recall of their construction, we will show how it can be adapted to differential modules over the field of Laurent series in one variable with complex coefficients. For such a differential module $\mathcal{M}$, we will present a formula relating Abbes and Saito’s construction to the differential forms occurring in the Levelt-Turrittin decomposition of $\mathcal{M}$. 