Harmonic functions in a domain with a small hole: the two-dimensional case

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The asymptotic behaviour of the solutions of boundary value problems in domains with small holes has been largely investigated by many authors with different approaches. In this seminar, we consider a Dirichlet problem for the Laplace operator in a bounded domain $\Omega^\epsilon$ of $\mathbb{R}^n$ containing the origin, where we remove a small set whose size is determined by a parameter $\epsilon$ and which collapses to 0 for $\epsilon = 0$. Then for $\epsilon \neq 0$ we denote the solution to such a problem by $u_\epsilon$. If $p \in \Omega^\epsilon$ and $p \neq 0$, then it makes sense to consider for $\epsilon \neq 0$ and ‘small’ the value of the solution $u_\epsilon$ at the point $p$. It is natural to ask what can be said on the map which takes $\epsilon$ small and positive to $u_\epsilon(p)$ around the degenerate value $\epsilon = 0$. One can try to answer to this question in several ways. By the approach proposed by Lanza de Cristoforis, one can show that, if $n \geq 3$, then there exist $\epsilon_p > 0$ and a real analytic function $U_p$ from $]-\epsilon_p, \epsilon_p[$ to $\mathbb{R}$ such that $u_\epsilon(p) = U_p[\epsilon]$ for all $\epsilon \in ]0, \epsilon_p[$, and one can then investigate the validity of such an equality for $\epsilon$ negative. After an introductory part on the case of dimension $n \geq 3$, we will turn to consider the two-dimensional case.

Based on joint work with M. Dalla Riva (CIDMA, Universidade de Aveiro).

Reference
