Finite Elements for Reissner-Mindlin plate bending problems

Carlo Lovadina

Abstract

We consider the plate bending problem in the framework of the Reissner-Mindlin theory. For a clamped plate, the problem can be written in variational form as follows.

Find \((\theta, w) \in (H^1_0(\Omega))^2 \times H^1_0(\Omega)\) :

\[
\int_{\Omega} C\varepsilon(\theta) : \varepsilon(\eta) + \mu t^{-2} \int_{\Omega} (\nabla w - \theta) : (\nabla v - \eta) = \int_{\Omega} gv,
\]

for every \((\eta, v) \in (H^1_0(\Omega))^2 \times H^1_0(\Omega)\). Here, \(\Omega\) is the plate mid-plane, \(t\) is the thickness parameter, \(\theta\) represents the fiber rotations, \(w\) the deflections, and \(g\) a given transversal load. Moreover, \(\varepsilon\) is the usual symmetric gradient operator, \(C\) is the tensor of bending moduli, and \(\mu\) is the shear modulus. Despite its simple formulation, the discretization by means of Finite Elements is not at all straightforward. The main difficulties which arise are connected to the following phenomena.

- **Shear locking effect.** It occurs for “small” plate thickness \(t\), when the Reissner-Mindlin model essentially converges to the Kirchhoff model. Shear locking is then caused by a poor approximation of the Kirchhoff constraint at the discrete level.

- **Spurious mode occurrence.** It refers to the occurrence of undesirable unphysical oscillations of the discrete solution. In the Reissner-Mindlin plate context, this phenomenon mainly occurs for the vertical displacement variable.

- **Boundary layer effects.** High Sobolev norms of the continuous solution to the Reissner-Mindlin equations are not bounded, uniformly in the plate thickness parameter \(t\), even for smooth data. This effect prevents the numerical schemes (especially of higher-order) to display optimal convergence rates on quasi-uniform meshes of realistic size.

In this talk we will focus on the first two issues mentioned above, and we will review the most popular strategies to overcome the troubles arising from them.