Extensions of the Kac-Stroock approximations

Let \( \{N(t), t \geq 0\} \) be a standard Poisson process and consider, for all \( n \in \mathbb{N} \) the process :

\[
x_n = \left\{ x_n(t) := \frac{1}{\sqrt{n}} \int_0^{nt} (-1)^{N(u)} du, \ t \in [0, T] \right\}.
\]

Stroock proves in 1982 that the processes \( x_n \) converge in law to a standard Brownian motion.

That is, if we consider \( (P^n) \) the image law of the process \( x_n \) in the Banach space \( C([0, T]) \) of continuous functions on \( [0, T] \), then \( (P^n) \) converges weakly, when \( n \) tends to infinity, towards the Wiener measure.

We find in the literature a lot of generalizations of the Stroock result in three directions :

– Modifying the processes \( x_n \) in order to obtain approximations of other Gaussian processes.

– Proving convergence in a stronger sense that the convergence in law in the space of continuous functions.

– Weakening the conditions of the approximating processes, that is, to find generalizations of the processes \( (-1)^{N(u)} \) that also converge to the Brownian motion.

In this talk we will analyze the processes considered by Stroock and we will recall the main results in this three directions. In particular, we will present and extension of the Kac-Stroock result using a Lévy process instead of Poisson process.