LAGRANGIAN REPRESENTATION FOR 1D HYPERBOLIC SYSTEMS OF CONSERVATION LAWS

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One of the key observations in Fluid Dynamics is that the fluid flow can be described from two different (and in some sense complementary) points of view: the Lagrangian point of view, (in which the trajectory in space-time of each single fluid particle is tracked) and the Eulerian point of view (in which one looks at fluid motion focusing on fixed locations in the space through which the fluid flows as time passes). Such key observation has been successfully applied to the analysis of some particular partial differential equations (among all, the transport equation, see for instance [3]), leading to important theoretical results.

In this talk, I will show that also the hyperbolic system of conservation laws in one space variable in its most general form:

$$\begin{cases} u_t + F(u)_x = 0, \\ u(0,x) = \bar{u}(x), \end{cases} \qquad u = u(t,x) \in \mathbb{R}^N,\ t \geq 0,\ x \in \mathbb{R},\ \text{Tot.Var.}(\bar{u}) \ll 1, \tag{1}$$

can be analyzed from a Lagrangian point of view. Here $F : \mathbb{R}^N \to \mathbb{R}^N$ is a generic smooth function, which is only assumed to be *strictly hyperbolic*, i.e. its differential $DF(u)$ has N distinct real eigenvalues in each point of its domain.

The well-posedness of the system (1) has already been proved (see for instance [1, 2, 4]). However several questions are still open, like the analysis of the fine structure of the solutions or the dependence of the solutions on the initial (or boundary) data. We think that our Lagrangian approach can lead to a deeper understanding of the behavior of the solutions to (1), thus casting new light on such open problems.

This is a joint work with Stefano Bianchini.

References

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