A system of two hyperbolic equations

\[ H_t + Q_x = 0 \]

conservation law

balance law \[ Q_t + [f(H, Q)]_x = g(H, Q) \]

\( H = \) density

\( Q = \) flow

Time \( t \in [0, +\infty] \)

Space \( x \in [0, L] \)
A system of two hyperbolic equations

\[ H_t + Q_x = 0 \]
\[ Q_t + [f(H, Q)]_x = g(H, Q) \]

\[ \begin{pmatrix} H_t \\ Q_t \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ f_H & f_Q \end{pmatrix} \begin{pmatrix} H_x \\ Q_x \end{pmatrix} = \begin{pmatrix} 0 \\ g(H, Q) \end{pmatrix} \]

\[ \begin{align*}
H &= \text{density} \\
Q &= \text{flow}
\end{align*} \]

\[ -\lambda_2 < 0 < \lambda_1 \]

Time \( t \in [0, +\infty) \)
Space \( x \in [0, L] \)
### Examples

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</table>
\[ Q(0, t) = Q_o(t) \]

\[ Q(L, t) = Q_L(t) \]

\[ H(x, t), \quad Q(x, t) \]

\[ H_t + Q_x = 0 \]

\[ Q_t + [f(H, Q)]_x = g(H, Q) \]

Control system with \( Q_o(t) \) and/or \( Q_L(t) \) as control inputs.

Stability and feedback stabilisation with \( H(0, t) \) and/or \( H(L, t) \) as measurements?
Case 1: \( g(H, Q) = 0 \)

\[
Q(0, t) = \varrho(t) \quad Q(L, t) = \varrho_L(t)
\]

\[
H(x, t), \quad Q(x, t)
\]

\[
H_t + Q_x = 0
\]

\[
Q_t + [f(H, Q)]_x = 0
\]

Any constant state \( H^*, Q^* \) can be a steady-state which is open-loop unstable! \( \rightleftharpoons \) need for feedback stabilization

Assume that \( H^* \) is the control set point.
Feedback control objective: system stabilisation + regulation + rejection of measurable disturbance.
Feedback control objective: system stabilisation + regulation + rejection of measurable disturbance.

\[ U_o(t) = Q_L(t) + k_P(H^* - H(0, t)) \]
Linearization

\[ \partial_t H + \partial_x Q = 0 \]

\[ \partial_t Q + \lambda_1 \lambda_2 \partial_x H + (\lambda_1 - \lambda_2) \partial_x Q = 0 \]

\[ \lambda_1, \lambda_2 > 0 \]

canonical

Riemann coordinates

\[ R_1 = Q - Q^* + \lambda_2 (H - H^*) , \]
\[ R_2 = Q - Q^* - \lambda_1 (H - H^*) , \]

\[ H = H^* + \frac{R_1 - R_2}{\lambda_1 + \lambda_2} , \]

\[ Q = Q^* + \frac{\lambda_1 R_1 + \lambda_2 R_2}{\lambda_1 + \lambda_2} \]

Boundary conditions for a constant disturbance \( Q_L(t) = Q^* \)

\[ \begin{pmatrix} R_1(t, 0) \\ R_2(t, L) \end{pmatrix} = \begin{pmatrix} 0 & k_1 \\ k_2 & 0 \end{pmatrix} \begin{pmatrix} R_1(t, L) \\ R_2(t, 0) \end{pmatrix} \]

\[ k_1 = \frac{k_P - \lambda_2}{k_P + \lambda_1} \]
\[ k_2 = -\frac{\lambda_1}{\lambda_2} \]
Closed-loop exponential stability

In the frequency domain, the closed loop system can be represented as a feedback delay system with characteristic equation:

$$e^{s\tau} - k_1 k_2 = 0,$$

where

$$\tau = \tau_1 + \tau_2$$

$$\tau_1 = \frac{L}{\lambda_1}$$

$$\tau_2 = \frac{L}{\lambda_2}$$

and

$$\sigma = -\frac{1}{\tau} \ln \left( \frac{1}{|k_1 k_2|} \right) < 0 \Leftrightarrow |k_1 k_2| < 1$$
Feedback control objective: system stabilisation + regulation + rejection of measurable disturbance.

\[ U_o(t) = Q_L(t) + k_P(H^* - H(0, t)) \]

Exponential Stability

\[ \implies k_P \text{ selected such that } |k_1 k_2| = \left| \left( \frac{k_P - \lambda_2}{k_P + \lambda_1} \right) \frac{\lambda_1}{\lambda_2} \right| < 1 \]
Feedback control objective: system stabilisation + regulation and attenuation of low frequency unknown load disturbances and perfect rejection of constant disturbance
Feedback control objective: system stabilisation + regulation and attenuation of low frequency unknown load disturbances and perfect rejection of constant disturbance

\[ Q_o(t) = Q_R + k_P(H^* - H(0, t)) + k_I \int_0^t (H^* - H(0, \tau)) \, d\tau \]

Proportional - Integral control law
Closed-loop exponential stability

**characteristic equation**

\[(e^{s\tau} - k_1 k_2)s + k_3(e^{s\tau} - k_2) = 0\]

\[k_1 = \frac{k_P - \lambda_2}{k_P + \lambda_1}\]

\[k_2 = -\frac{\lambda_1}{\lambda_2}\]

\[k_3 = \frac{k_I}{k_P + \lambda_1}\]

root locus
for \(|k_1 k_2| < 1\)
and \(k_3 : 0 \rightarrow +\infty\)
Control input: \( Q(0, t) = U_o(t) \)

Measurement: \( H(0, t) \)

Unknown disturbance input: \( Q(L, t) = Q_L(t) \)

Proportional - Integral control

\[
Q_o(t) = Q_R + k_P (H^* - H(0, t)) + k_I \int_0^t (H^* - H(0, \tau)) d\tau
\]

Exponential Stability

\[ k_P \text{ selected such that } |k_1 k_2| = \left| \frac{k_P - \lambda_2}{k_P + \lambda_1} \right| \frac{\lambda_1}{\lambda_2} < 1 \]

and \( k_I > 0 \) sufficiently small
What about « non-local » control ?

\[ U_o(t) = Q_L(t) + k_P(H^* - H(L, t)) \]

For simplicity we consider the special case \( \lambda_1 = \lambda_2 = \lambda \)

\[ \partial_t H + \partial_x Q = 0, \]
\[ \partial_t Q + \lambda^2 \partial_x H = 0. \]
Closed-loop exponential stability?

\[ U_o(t) = Q_L(t) + k_P(H^* - H(L,t)) \]

Unstable for all \( k_P \)!
\[ Q(0, t) = U_o(t) \]

\[ H(x, t), \quad Q(x, t) \]

\[ U_o(t) = Q_L(t) + k_P(H^* - \hat{H}(0, t)) \]

on-line estimate
control input \[ Q(0, t) = U_o(t) \]

\[ \text{measurement} \quad H(L, t) \]

disturbance input \[ Q(L, t) = Q_L(t) \]

\[
U_o(t) = Q_L(t) + k_P(H^* - \hat{H}(0, t))
\]

Observer

\[
\partial_t \hat{H} + \partial_x \hat{Q} = 0,
\]

\[
\partial_t \hat{Q} + \lambda^2 \partial_x \hat{H} = 0.
\]

\[
\hat{Q}(0, t) = Q_0(t) = Q_L(t) + k_P(H^* - \hat{H}(0, t))
\]

\[
\hat{Q}(L, t) = Q_L(t) + \nu(H(t, L) - \hat{H}(L, t))
\]
\[ Q(0, t) = U_o(t) \]

\[ Q(L, t) = Q_L(t) \]

Control input

Measurement \( H(L, t) \)

Disturbance input

\[ H(x, t), \quad Q(x, t) \]

Observer tuning parameters

\[ U_o(t) = Q_L(t) + k_P(H^* - \hat{H}(0, t)) \]

\begin{align*}
\hat{Q}(0, t) &= Q_0(t) = Q_L(t) + k_P(H^* - \hat{H}(0, t)) \\
\hat{Q}(L, t) &= Q_L(t) + \nu(H(t, L) - \hat{H}(L, t))
\end{align*}
Closed-loop exponential stability

(for simplicity, $\lambda_1 = \lambda_2 = \lambda$, no loss of generality)

\[ \begin{pmatrix} R_1(t, 0) \\ R_2(t, L) \\ \tilde{R}_1(t, 0) \\ \tilde{R}_2(t, L) \end{pmatrix} = \begin{pmatrix} 0 & \frac{k_P - \lambda}{k_P + \lambda} & 0 & -\frac{2k_P}{k_P + \lambda} \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ \frac{2\lambda}{\lambda + \nu} & 0 & \frac{\nu - \lambda}{\nu + \lambda} & 0 \end{pmatrix} K \begin{pmatrix} R_1(t, L) \\ R_2(t, 0) \\ \tilde{R}_1(t, L) \\ \tilde{R}_2(t, 0) \end{pmatrix} \]

**Boundary conditions in Riemann coordinates**

\[ \begin{pmatrix} R_1(t, 0) \\ R_2(t, L) \end{pmatrix} = \begin{pmatrix} e^{R_1(t, 0)} \\ e^{R_2(t, L)} \end{pmatrix} \]

**tuning parameters**

\[
\begin{align*}
\nu &= \lambda \text{ (dead-beat observer)} \\
0 &< k_P < \lambda
\end{align*}
\]

\[ \Rightarrow \rho(|K|) < 1 \quad \Rightarrow \quad \text{Exponential Stability} \]
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

\[ H_j = \text{density} \quad \partial_t H_j + \partial_x Q_j = 0, \quad j = 1, \ldots, n \]

\[ Q_j = \text{flow} \quad \partial_t Q_j + f_j(H_j, Q_j) = 0, \]

\[ Q_i(t, 0) = U_i(t) + \sum D_k(t), \quad i = 1, \ldots, n, \]

boundary conditions

\[ Q_i(t, L) = \sum U_j(t) + \sum D_k(t), \quad i = 1, \ldots, n. \]
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

\[ \partial_t H_j + \partial_x Q_j = 0, \quad j = 1, \ldots, n \]
\[ \partial_t Q_j + f_j(H_j, Q_j) = 0, \]
\[ Q_i(t, 0) = U_i(t) + \sum D_k(t), \quad i = 1, \ldots, n, \]
\[ Q_i(t, L) = \sum U_j(t) + \sum D_k(t), \quad i = 1, \ldots, n. \]
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

The network has an infinity of positive steady states which are not open-loop asymptotically stable. In order to stabilize the network, with attenuation of load disturbances, each control input is endowed with a PI feedback control of the form:

\[ U_i(t) = U_R + k_{PI}(H_i^* - H_i(t, 0)) + k_{II} \int_0^t (H_i^* - H_i(\eta, 0)) d\eta \]
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

\[ \frac{G_i(s)}{Q_i(s, L) - Q^*} = \frac{s(\lambda_i k_i + \lambda_{n+i}) + c_i (\lambda_i + \lambda_{n+i})}{(e^{s\tau_i} - k_i k_{n+i})s + c_i(e^{s\tau_i} - k_{n+i})} e^{sL} \]

\[ k_i = \frac{k_{Pi} - \lambda_{n+i}}{k_{Pi} + \lambda_i}, \quad k_{n+i} = -\frac{\lambda_{n+i}}{\lambda_i}, \]

\[ c_i = \frac{k_{Li}}{k_{Pi} + \lambda_i}, \quad \tau_i = \frac{L}{\lambda_i} + \frac{L}{\lambda_{n+i}} \]
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

Transfer matrix between $D_i$ and $U_i$ for the global closed-loop system

$$H(s) = \sum_{i=0}^{p-1} (G(s)A_L)^i (G(s)B_L - B_0)$$

- $G(s) = \text{diag}\{G_1(s), \ldots, G_n(s)\}$
- $p$ is the length of the longest path in the network
- $A_L, B_0, B_L$ are matrices reflecting the network topology, with $A_L^p = 0$
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

Transfer matrix between $D_i$ and $U_i$ for the global closed-loop system

$$H(s) = \sum_{i=0}^{p-1} (G(s)A_L)^i (G(s)B_L - B_0)$$

$$H(s) = \begin{pmatrix}
G_1 & -G_1G_3 & G_1G_3 & G_1G_4 & G_1G_3G_5 \\
-1 & G_2 & 0 & 0 & 0 \\
0 & -G_3 & G_3 & 0 & G_3G_5 \\
0 & 0 & 0 & G_4 & 0 \\
0 & 0 & 0 & 0 & G_5
\end{pmatrix}$$
Networks of density-flow systems

e.g. electrical networks, hydraulic networks, gas pipe networks, blood networks, road traffic networks etc ...

Transfer matrix between $D_i$ and $U_i$ for the global closed-loop system

$$H(s) = \sum_{i=0}^{p-1} (G(s)A_L)^i (G(s)B_L - B_0)$$

The poles of $H(s)$ are given by the collection of the poles of the individual scalar transfer functions $G_i(s)$
The poles of the closed-loop system lie in the LHP \textit{if and only if} the control tuning parameters are selected such that ......

- when $\lambda_i < \lambda_{n+i}$,
  \[
  k_{P_i} > 0 \quad \text{and} \quad k_{I_i} > 0 \quad \text{or} \quad k_{P_i} < -\frac{2\lambda_i \lambda_{n+i}}{\lambda_{n+i} - \lambda_i} \quad \text{and} \quad k_{I_i} < 0;
  \]

- when $\lambda_i = \lambda_{n+i}$, $k_{P_i} > 0$ and $k_{I_i} > 0$;

- when $\lambda_i > \lambda_{n+i}$,
  \[
  0 < k_{P_i} < \frac{2\lambda_i \lambda_{n+i}}{\lambda_i - \lambda_{n+i}} \quad \text{and} \quad 0 < k_{I_i} < \omega_i \left( \frac{2k_P + \lambda_i - \lambda_{n+i}}{\lambda_i^2 - \lambda_{n+i}^2} \right) \lambda_i \lambda_{n+i} \sin(\omega_i \tau_i)
  \]
  where $\omega_i$ is the smallest positive $\omega$ such that

  \[
  \cos(\omega \tau_i) = \frac{\lambda_{n+i}^2 (k_{P_i} + \lambda_i) + \lambda_i^2 (k_{P_i} - \lambda_{n+i})}{\lambda_i \lambda_{n+i} (\lambda_{n+i} - \lambda_i - 2k_{P_i})}.
  \]

\textit{Bastin, Coron, Tamasoiu, Automatica 2014}
Remark on exponential stability

Conditions such as “poles in the complex LHP” or $|k_1 k_2| < 1$ or $\rho(|K|) < 1$, guarantee the global exponential convergence to zero of the $L^p$-norm:

$$|H(x, t) - H^*|_{L^p} + |Q(x, t) - Q^*|_{L^p}$$

for all $p > 1$. 
Global exponential stability of a linearized system of conservation laws for the $L^p$ norm ($p>1$)

Local exponential stability of the steady-state of the corresponding nonlinear system for the $H^p$ norm ($p>1$) with adapted compatibility conditions on the initial data

Coron, Bastin, d’Andréa-Novel, SIAM Journal of Control, 2008 (special case $p=2$)
In many practical applications, the source term $g(H, Q)$ has dissipativity properties such that the open loop system is now asymptotically stable!
$Q(0, t) = U_o(t)$

$H(x, t), \quad Q(x, t)$

$H_t + Q_x = 0$

$Q_t + [f(H, Q)]_x = g(H, Q)$

**Case 2:** $g(H, Q) \neq 0$

**Steady-state**

$Q^* = \text{constant} \quad H^*(x) \text{ s.t.} \quad \frac{d}{dx} f(H^*(x), Q^*) = g(H^*(x), Q^*)$

**Control law**

$U_o(t) = Q_L(t) + k_P(H^*(0) - H(0, t))$
Linearisation in Riemann coordinates

\[ \partial_t R_1 + \lambda_1(x) \partial_x R_1 = a(x) R_2 \]
\[ \partial_t R_2 - \lambda_2(x) \partial_x R_2 = b(x) R_1 \]

\[ R_1(t, 0) = k_1 R_2(t, 0) \]
\[ R_2(t, L) = k_2 R_1(t, L) \]

\[ k_1 = \frac{k_P - \lambda_2(0)}{k_P + \lambda_1(0)} \]
\[ k_2 = -\frac{\lambda_2(L)}{\lambda_1(L)} \]

Lyapunov function

\[ V(t) = \int_0^L \left[ q_1(x) R_1^2(t, x) + q_2(x) R_2^2(t, x) \right] dx \]

\[ \exists q_1(x), q_2(x) \text{ such that } V(t) \text{ is a Lyapunov function if} \]

1) The solution of the ODE \[ \frac{d\eta}{dx} = \left[ \frac{a(x)}{\lambda_1(x)} + \frac{b(x)}{\lambda_2(x)} [\eta(x)]^2 \right], \eta(0) = 0 \text{ is defined on } [0, L] \]
2) \[ \left| \frac{\lambda_2(L)}{\lambda_1(L)} \right| \geq \eta(L) \]

Then stability if \[ k_1^2 \leq \frac{\lambda_2(0) q_2(0)}{\lambda_1(0) q_1(0)} \]
Real-life application: Control of navigable rivers

Meuse river (Belgium)
Real-life application: level control in navigable rivers. Meuse river (Belgium).

« Haute Meuse »
A navigable river is a chain of pools separated by hydraulic gates (and sluices for the boats !)
Weir of La Plante

4 automated hydraulic gates used for level regulation

sluice
Each automated control gate can be operated in both overflow or underflow mode.
Model: Saint-Venant equations

\[
\begin{align*}
\partial_t \left( \frac{A_i(H_i)}{V_i} \right) + \partial_x \left( \frac{A_i(H_i)V_i}{2V_i^2 - gH_i} \right) &= \begin{pmatrix} 0 \\ g[S_i - S_f(H_i, V_i)] \end{pmatrix} \\
&\quad i = 1, \ldots, n
\end{align*}
\]

Boundary conditions

1) Conservation of flows \( A_i(H_i(t, L))V_i(t, L) = A_{i+1}(H_{i+1}(t, 0))V_{i+1}(t, 0) \quad i = 1, \ldots, n - 1 \)

2) Gate models \( A_i(H_i(t, L))V_i(t, L) = k_G \sqrt{[H_i(t, L) - u_i(t)]^3} \quad i = 1, \ldots, n \)

3) Input flow \( A_1(H_1(t, 0))V_1(t, 0) = Q_0(t) = \text{disturbance input in navigable rivers} \)

\( \text{or control input in irrigation networks} \)
Cross-section: the river bathymetry has been recorded by using a swath sonar system.
Numerical simulation

The Saint Venant equations are integrated numerically using a standard Godunov scheme with a spatial step size = 1 m and a time step = 1 sec
Control design issues

• Local or non local control ?
• Choice of the set point
• Choice of the time step
Local versus non-local feedback control

Local

Non-local

PI controller

Control action

Level measurement

PI controller

Control action

Level measurement
The effect of control action is delayed which limits the achievable performance. Used in irrigation networks for water saving.
Floods of Meuse river

Flow rates from 30 m³/sec (summer) up to 1500 m³/sec (winter)
Steady-state profile computed with the model

High flow rate
\[ \approx 600 \text{ m}^3/\text{sec} \]
Steady-state profile computed with the model

Low flow rate
≈ 60 m³/sec
Steady-state profile computed with the model
In order to have a sufficient water depth for a wide range of flow rates, the set point $H^*$ must depend on the flow rate value!

At the bottom of the pool, we have the counterintuitive observation that the level is lower when the flow rate is higher!
Control law

\[ U_i(t) = U_R + k_{P_i}(H_i^* - H_i(t, L)) + k_{I_i} \int_0^t (H_i^* - H_i(\eta, L)) d\eta \]

*Proportional*  
*Integral*

**arbitrary constant reference value**

\[ U_i(t) \] = level of the gate = control action

The parameters \( k_P \) and \( k_I \) are tuned to have, in simulation, a closed loop unit step response similar to the open loop.
Control law

\[ U_i(t) = U_R + k_{Pi}(H_i^* - H_i(t, L)) + k_{Ii} \int_0^t (H_i^* - H_i(\eta, L)) d\eta \]

arbitrary constant reference value

Proportional Integral

Digital incremental implementation

\[ U_i(t) = U_i(t - \Delta t) + k_{Pi}(e_i(t) - e_i(t - \Delta t)) + k_{Ii}e_i(t) \]

\[ e_i(t) = H_i^*(t) - H_i(t, L) \]

set point filtered level data
(time step 1 min)

\( \Delta t = 20 \text{ min} \)

robustness against uncertainties on the relationship between gate level and flow rate
$\Delta t = 20 \text{ min}$

Why?

unit step response time (usrt)
computed with the Saint Venant model

flow rate

$\text{Tailfer Weir (4 gates)}$

$\text{Hun Weir (3 gates)}$
unit step response time computed with the Saint Venant model

$\Delta t = 20\ \text{min}$

because unit step response time $\approx 5 \ldots 10\ \Delta t$
Unit step response time (usrt) computed with the Saint Venant model

Why?
Because ....

Removable micro hydro-electric power plant
Control display
Experimental result (Dinant)

16 to 23 October 2012

- Water level
- Change of set point
- High flow rate
- Low flow rate
- Gate levels = control action

Meters above sea level
Stability and Boundary Stabilization of 1-D Hyperbolic Systems

Georges Bastin & Jean-Michel Coron

Thank you!
Thank you!