Number Theory 1

(Algebraic Number Theory)

Fall Term 2007/08

Teacher: F. Baldassarri

Textbook.

Gerald J. Janusz : *Algebraic Number Fields*, Graduate Studies in Mathematics Volume 7 American Mathematical Society 1996.

Program.

We plan to follow faithfully the exposition in the first two chapters of the textbook. The topics to be covered will be:

1. Short review of localization and integral dependence.
2. Discrete valuation rings and Dedekind rings. UFD property.
3. Fractional ideals and the class group.
4. Normal Basis Theorem and Hilbert's Theorem 90.
5. Extensions of Dedekind rings.
6. Ramification and the discriminant.
7. Norms of ideals.
8. Explicit determination of ring of integers.
9. Cyclotomic fields. Gauss sums.
10. Quadratic fields, quadratic reciprocity law.
11. Minkowski Theory (finiteness of class number and the unit theorem, the regulator).
12. Valuations and completions. Local fields.
13. Extensions of local fields. Decomposition and inertia groups.
14. Product formula.

Prerequisites. Some notions of the theory of fields (splitting fields, separability, normality, the theorem of the primitive element). The principles of Galois theory. Some elementary notions of commutative algebra (rings, modules, localization, integral dependence, tensor products,..).

Schedule. Three 90 minute sessions a week for eight weeks during the Fall Term. Classes will be taught in English. Starting date and venue of the course to be announced.

Francesco Baldassarri