Number Theory 2

Adrian Iovita

December 10, 2008

The main objective of this course is the study of local fields and of their Galois groups. More precisely if K is a local field of characteristic $(0, p)$, where p is a rational prime, we denote by $\overline{K}$ an algebraic closure of K and by G_K the Galois group of $\overline{K}$ over K. It is a non-abelian, compact group which has a very interesting ramification filtration. We will study the continuous representations of G_K on finite-dimensional $\mathbb{Q}_p$-vestor spaces, called *Galois representations* and their classification by Fontaine's theory. Such Galois representations arise naturally as étale cohomology groups of algebraic varieties over K and their study is one of the central goals of Arithmetic Geometry.

The course will initiate with an introduction to local fields by F. Baldassarri. He will review some generalities on local fields and Galois extensions of such, based on the first 3 chapters of [Se].

The course will be integrated by some lectures on local fields by Professor Sergei Vostokov, from the University of St. Petersburg. He will cover some of the following topics:

- Conics and p-adic Numbers (Theorem of existence of a rational point on a conic using residue symbol in p-adic number field)
- Local fields - basic definitions and theorems.
- The Kronecker-Weber Theorem for a p-adic field.
- The Lubin-Tate theory as extension of the Kronecker-Weber Theorem.
- Artin-Hasse function and its role in arithmetic of local fields and formal groups.

References

[1] Jean-Pierre Serre. *Corps Locaux*, Actualités scientifiques et industrielles Vol. 1296. Hermann, Paris 1989.

[2] Ivan Fesenko and Sergei Vostokov *Local Fields and their Extensions*, Mathematical Monographs Vol. 121, Second Edition, American Mathematical Society 2002.