Risk Management and Stress Testing

From Theory to Practice

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What did I learn from Wolfgang Runggaldier?

A lot of things…mainly the love for knowledge
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Goal of the talk

I will not give

• Theoretical results

I will give you

• Some (I hope) interesting problems arising from practice
• Some ideas to solve them
• Some references
# of Dimensions, Positions and Risk factors in a real portfolio

- **10,000** of different instruments
- **100,000** deal (positions)
- **1,000** risk factors
- Portfolio tree with **500** (intermediates or terminal) nodes
- 2 years, e.g. **500** cases of daily data in order to estimate risk measures
- **15** categorical variables for each single position in order to get reporting, that is one has to get risk measure for each way of clustering by these variables
Case 1. The *gaussian VaR*: an alternative perspective

The problem
- The VaR is simply given by
  \[ \text{VaR}_{PTF} = V_{PTF} \cdot \sigma_{PTF} \cdot z_{1-\alpha} \cdot \sqrt{h} \]
- Many people say “VaR? it is just a quantile …”
- But
  - Do we really know volatility \( \sigma \)?
  - Do we really know the value \( V \)?
  - How to deal with missing data, outliers, unlisted financial instrument?
  - How to deal actually with working days and calendar days?
Case 1. The *gaussian VaR*: an alternative perspective

Some ideas

- To look at VaR (the same for other indicators of risk or returns) as **process**, not only as a one step **algorithm**
- For example, to use *DFD, Data flow diagram*, or to use the *PERT graph* from Operations Research
- In this way, one can manage the weakness of the process, missing data, failure of computations and so on
Case 2. Quantile (VaR) estimation in simulation context

The problem

- *Historical simulation* is in recent years receiving attention from the central banks, for the regulatory capital.
- Given the sample of returns \( r = (r_1, r_2, \ldots, r_N) \) of the instrument/portfolio, one could simply estimate the quantile by the naif quantile estimator

\[
VaR = r_{[(1-\alpha)N]}
\]

- The notation ( ) stands for the order statistics rearranged sample
- But, due to nature of this estimator
  - the “history” of VaR could become a bizarre, unbelievable
  - The variability is very high
Case 2. *Quantile (VaR) estimation in simulation context*

The problem

- In the graph, a VaR history of an instrument with perfectly constant gaussian return, volatility 1% (the “true” 99% VaR is -2.32%)
- The market is stable, but the VaR estimates change in a strange way, because some old returns go out from the sample, some recent enter in the sample
Case 2. Quantile (VaR) estimation in simulation context

Some ideas

- To check under control the variability of the estimates, for example by the results from the order statistics theory, when we know that asymptotically the order statistics (except the min and the MAX) have a gaussian distribution with variance. So, we can compute some confidence interval for the estimator

\[
E(\hat{x}_p) = x_p - \frac{p(1 - p)f'(x_p)}{2(n + 2)f^2(x_p)} + O(1/n^2);
\]

\[
\text{Var}(\hat{x}_p) = \frac{p(1 - p)}{(n + 2)f^2(x_p)} + O(1/n^2).
\]  

- To enhance the estimation in a bias-variance trade-off, e.g. by using the L-estimators, linear combination of order statistics. We have
  - Rectangular L-estimators, equally weighted
  - Some more sophisticated shapes, such as the Harrel Davis (HD) estimator

Figure 1: Weights for HD estimator
\((S = 100, \alpha = 0.95)\)
Case 2. Quantile (VaR) estimation in simulation context

Some references

Calculating Quantile-based Risk Analytics with $L$-estimators

Helmut Mausser

Proceedings of the 2002 Winter Simulation Conference

TWO-PHASE QUANTILE ESTIMATION

E. Jack Chen

BASF Corporation
3000 Continental Drive - North
Mount Olive, NJ 07828-1234, U.S.A.
Case 3. *Ex-ante* vs. *Ex-post* gaussian VaR in portfolio management

The problem

- The real focus of risk managers and asset managers is in the portfolio volatility and risk, not on the single instruments.
- The gaussian approach is widely appreciated by asset managers (funds, private banking, ...) because of the simplicity of *volatility / correlation* principles.
- All techniques are base upon historical data, but we can use at least 2 techniques:
  - *Ex-ante*. The volatility $\sigma_{\text{PTF}}$ is computed from the present weights $w_i$, the correlations $\rho_{ij}$ and the assets volatilities $\sigma_i$.
  - *Ex-post*. We compute the portfolio returns $R_t,\text{PTF}$ and from it we estimate with usual methods the $\sigma_{\text{PTF}}$. This techniques is also known as *portfolio-normal*.
Case 3. *Ex-ante* vs. *Ex-post* gaussian VaR in portfolio management

Some ideas

- What techniques work better? It depends on the type of portfolio. For standard low and medium risk funds, the *ex-ante* is good. For hedge and flexible funds, the *ex-post* is often better.
- Remind that the portfolio returns are not so easy because of the flows $F_t$ in the portfolio: each day the customer can put or take money!

$$ R_t = \frac{V_t - F_t - V_{t-1}}{V_{t-1}} $$

- To clean data (e.g. to intercept the $F_t$) is often quite difficult, because one has to scan the whole history and apply a filter depending on the class of operation in the portfolio. Not all the cash flows have to be dropped out: dividends, coupons, …
Case 3. *Ex-ante* vs. *Ex-post* gaussian VaR in portfolio management

Some references

- The GIPS standard for the performance presentations, and the Italian Release, arranged by IIPC
Case 4. Marginal Full Evaluation in simulation approaches

The problem

- In a simulation (historical or montecarlo) approach to VaR, the best way to evaluate the P&L over the difference scenarios is the full evaluation, that is for each scenario \( t = 1, \ldots, T \) to price the position by the suited algorithms.
- Let \( \phi(m_1, \ldots, m_k) \) be the pricing functions depending on the market parameters (we omit the dependence from instrument data such as strike for simplicity).
- If we have simulated shocks \( \Delta_{1,t}, \ldots, \Delta_{k,t} \), the full evaluation (in a strict sense) gives this rule for the global \( P&L_G \)

\[
P & L = \phi(m_1 + \Delta_{1,t}, \ldots, m_k + \Delta_{K,t}) - \phi(m_1, \ldots, m_k)
\]

- In risk management internal process, we need to decompose risk in its contributions, in a multifactorial approach. The risk managers require a risk view clustering by the different \( k \) market parameters, type of risks (delta, vega, rho).
- How to have a both conjoint and marginal coherent view of risks?
Case 4. *Marginal Full Evaluation* in simulation approaches

Some ideas

- To solve the above requirement, we apply single marginal shocks, and we define (it is an *approximation*) the global P&L as the sum of the marginal P&L

\[
P & L_k = \phi(m_1, \ldots, m_k + \Delta_{k,t}, m_K) - \phi(m_1, \ldots, m_k)
\]

\[
P & L_G \equiv \sum_k P & L_k
\]

- What is the insight of the idea? We recall that the exact full evaluation difference (for smooth functions $\phi$) may be written ad an infinite taylor edspansion, where use ude the gradient, the hessian and so on

- The *MFE* uses all the “pure” derivatives and loses the mixte derivatives

- Advantages
  - We can give an *additive* decomposition, very useful in practice
  - By only one Database table (the $P&L_{k,t,i}$ segmented by scenario t, instrument I, risk factor k) we can buld every kind of risk measure by aggregation, sorting, and linear operators
Case 4. *Marginal Full Evaluation* in simulation approaches

Some references

- How works the approximation for the instrument global P&L?
- Some numerical studies (Bonollo & Marinopiccoli, SCO 2005, Bressanone) give good results. For standard and “soft” exotic options, the MFE approximates better than the delta and delta-gamma techniques.
Case 5. *Interpolation & Grids*

**The problem**

- We use the same notation of the previous case. Suppose:
  - Scenarios $T = 500$
  - Instrument $I = 10,000$
  - Average number of risk factors per instrument $K = 3$

- In a marginal full evaluation approach, it means that each day we have:
  - To **run** $T \times I \times K = 15$ million of pricing $\phi$, some of them are montecarlo pricing
  - To **store** the results (P&L, PV, reference data) related to 15 mln of “objects”.

- For **audit**, **backtesting** and central bank **compliance**, we have to store the results and the intermediate information for at least 250 days
Case 5. Interpolation & Grids

Some ideas

- We can use some deterministic $G$ fixed shocks, with a different granularity, for the different classes of market parameters/risk factors
- Example
  - Level of underlying (delta risk): 15 shocks
  - Level of volatility (vega): 9 shocks
  - Level of interest rates (rho): 9 shocks
- Then we run the pricing function only for the points of the grid
- Finally we approximate the P&L, due to simulated scenarios (montecarlo or historical shocks) by interpolation: linear, bilinear, ..
- Plus and minus
  - Approximation error, to be summed to the Marginal FE error
  - Computational time savings: we can reduce between 10 and 50 the computation effort and space: $G << T$
Case 6. Risk decomposition, a data model for risk factors

Some references

- Bonollo, AMASES Conference, Florence 2006

Simulation of Fixed-income Portfolios Using Grids

Adnan Chishti

We consider the application of low-dimensional grids to the simulation of complex fixed-income portfolios. We describe the grid simulation methodology and present a rigorous testing framework to measure and decompose the approximation errors. The methodology is illustrated by a case study of a callable bond portfolio. We demonstrate that grids provide reasonably accurate non-parametric VaR estimates while substantially reducing the computational time required.
Case 6. Risk decomposition, a *data model* for risk factors

The problem

- What does “exotic derivatives” mean?
- One instrument may be exotic due to three classes of sophistication
  - Pay off formula: double strike, double barrier, …
  - Underlying: form one underlying to linear basket to nonlinear (warrants of, rainbow, ...) basket
  - Market data Fixing: the market data are taken in the average, the worst over time, …
- So, an asian option is a in exotic only in the average fixing, a rainbow option in exotic in the underlying, …
- When one develops pricing and risk methodologies for himself, one can thing to the INPUTS of pricing model as a FLAT structure.
- In software industry, with actual size problems, one has to optimize the data structure, splitting payoff, underlying, fixing information, market parameters, final user parameters and so on
Case 6. Risk decomposition, a data model for risk factors

Some ideas

- The E-R (Entity-relationships) model is a technique developed by a mathematician in the first ’70. Now, all databases (DB2, Oracle, SQL, Access) store the data following this elegant technique based on set theory, non-redundancy, injective functions.
- In my experience with young colleagues and stages, the software development without knowledge of data structure is often useless.
Case 7. Stress Testing and *Principal Components Analysis*

**The problem**

- There is an important gap between **front office** software systems and **risk management** requirements
  - The front office systems must guarantee the position keeping by exact (given the model) pricing. In doing that, a **large numbers** of market parameters are used, and an high level of **granularity** is the best practices. For example, the volatility surfaces are widely used in the practice, and a single surface may have 500 points in maturity/moneyness axis
  - The risk manager must have a **strategic view** of risk, possibly related to macroeconomic models. He is not interested to the single point in the vol surface, but to questions such as “*How will the volatility moves? How do the vols impact on the P&L?*”
Case 7. Stress Testing and Principal Components Analysis

Some ideas

• The PCA has been often used to model the term structure of interest rates movements. The first two components usually explain more than 90% of the dynamics of the whole structure, represented by 15-20 time buckets.

• This intuitive technique may be extended to more sophisticated parameters, volatility surfaces.

• We discover then the movements of the surface may be summarized by two components.

• Application: when the risk managers design (EVT, historical, subjective) stress scenarios, he can efficiently work on a small number of components.
Case 7. Stress Testing and *Principal Components Analysis*

ATM volatility PCA

ZC rate PCA