Projections and Biorthogonality *

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Timetable: 24 hours, Tuesday 14.00-16.00, Wednesday 10.00-12.00. Torre Archimede, Aula 2BC/30 (on March 16, 2010 and March 23, 2010 is held in Aula 2BC/60). First lesson on February 16, 2010.

Course requirements: No special prerequisite is needed except the usual concepts concerning linear algebra and vector spaces.

Examination and grading: Grading is based on homeworks or a written examination or both.

Aim: These lectures are intended to students and researchers in pure and applied mathematics, in numerical analysis, and in scientific computing.

Course contents:

Operator equations in infinite dimensional vector spaces are important in pure and applied mathematics.

Let $f \in E$, $b \in F$, where E and F are vector spaces, and $A : E \longmapsto F$ be a linear operator. We consider the linear operator equation

$$Af = b.$$

Three different problems, by increasing order of difficulty, can be treated

- the direct problem: given A and f, compute b. For example, the computation of a definite integral,
- the inverse problem : given A and b, compute f. For example, the solution of a system of linear equations,
- the identification problem : given f and b, compute A. For example, the approximation of functions.

Usually, the solution of such problems poses quite serious difficulties or is even impossible. Then, the original problem has to be replaced by an approximated one obtained by projection into a finite dimensional space.

The first part of these lectures will be devoted to the theory of projection methods. Application to the solution of large systems of linear equations will be discussed. The notion of biorthogonality, introduced by Banach in 1932, is closely related to projections.

The second part of these lectures will present its theory and its implementation, and give some of its applications.

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1. Projections
   (a) The setting
   (b) Extrapolation methods
   (c) Best approximation
   (d) Recursive projection
   (e) The method of moments of Vorobyev
   (f) Projection methods for systems of equations
      i. Richardson projection
      ii. Krylov subspace method
      iii. Lanczos method
2. Biorthogonality
   (a) The concept of biorthogonality
   (b) The general interpolation problem
   (c) Biorthogonalization processes
   (d) Recurrences for biorthogonality
   (e) Applications
      i. Fredholm equation
      ii. Padé–type approximants
      iii. Biorthogonal polynomials
      iv. Statistics and least squares

References

[1] C. Brezinski, *Padé–Type Approximation and General Orthogonal Polynomials*, ISNM, vol. 50, Birkhäuser–Verlag, Basel, 1980.

[2] C. Brezinski, *Biorthogonality and its Applications to Numerical Analysis*, Marcel Dekker, New York, 1992.

[3] C. Brezinski, *Projection Methods for Systems of Equations*, North-Holland, Amsterdam, 1997.

[4] C. Brezinski, M. Redivo-Zaglia, *Extrapolation Methods. Theory and Practice*, North–Holland, Amsterdam, 1991.

[5] A. Galántai, *Projectors and Projection Methods*, Kluwer Academic Publishers, Boston, 2004.

[6] N.S. Kurpel', *Projection-Iterative Methods for Solution of Operator Equations*, American Mathematical Society, Providence, Rhode Island, 1976.

[7] Yu.V. Vorobyev, *Methods of Moments in Applied Mathematics*, Gordon and Breach, New York, 1965.