Crystallographic groups

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Calendario: 15 ore, Torre Archimede, nei giorni
10 maggio 2010 ore 14.30-16.30, Aula 2BC/30
11 maggio 2010 ore 14.30-16.30, Aula 2AB/45
12 maggio 2010 ore 9.30-11.00, Aula 2BC/30
13 maggio 2010 ore 11.30-13.00, Aula 2BC/30
17 maggio 2010 ore 14.30-16.30, Aula 2BC/30
18 maggio 2010 ore 14.30-16.30, Aula 2AB/45
19 maggio 2010 ore 10.30-12.30, Aula 2BC/30
20 maggio 2010 ore 14.30-16.30, Aula 2BC/30

Prerequisiti: Elementary group theory, including semidirect products, actions of groups on sets, and the basics of nilpotent and soluble groups. Isometries of Euclidean affine space.

Tipologia di esame: Discussione orale.

SSD: MAT/02, MAT/07

Topics: If G is a discrete subgroup of the group of isometries Iso(R^n) of n-dimensional Euclidean space, then the translation subgroup T of G is isomorphic to Z^d for some $d \leq n$. In the case that T has maximal rank, i.e. that it is isomorphic to Z^n, we say that G is an n-dimensional crystallographic group (or space group). Threedimensional crystallographic groups were classified at the end of the nineteenth century independently by Barlow, Federov, and Schönfliess, and there are 230 nonisomorphic such groups.

If two discrete subgroups G and H of Iso(R^n) are isomorphic, it may nevertheless happen that they have a completely different geometric action on R^n. For example, both a rotation of angle π and a reflection in a line generate a cyclic subgroup of order 2 in Iso(R^2), but their geometric effect is not the same. Fortunately, a fundamental theorem of Bieberbach (1912) guarantees that this cannot happen with crystallographic groups of any dimension. Bieberbach himself and Frobenius are responsible for the introduction of algebraic methods in the theory of crystallographic groups at the beginning of the twentieth century, in contrast with the geometric approach followed by their predecessors. This led to the algebraic proof of the classification of crystallographic groups in dimensions 2 and 3, given by Burckhardt. Also, Bieberbach proved that the number of isomorphism classes of crystallographic groups is finite for every dimension. In 1948, Zassenhaus developed an algorithm for the determination of crystallographic groups, which was later used to obtain a computer-aided classification of these groups in dimension 4.

Recently, in a 2001 paper, Conway, Delgado Friedrichs, Huson, and Thurston provided a new scheme for the classification of three-dimensional crystallographic groups. It is based on obtaining the groups as fibrations over the plane crystallographic groups, whenever this is possible. There are 35 groups for which it is not, and these are called the *irreducible groups*. These groups are determined by studying the subgroup generated by the elements of order 3, for which it turns out that there are only two possibilities. The first one gives rise to 27 groups, and the second, to 8 more.

The goal of this PhD course is to give an exposition of the general algebraic theory of crystallographic groups in any dimension n, with special emphasis in the cases n=2 and n=3. We will also give the details of the scheme of Conway et al for the classification of three-dimensional crystallographic groups, and see how the different groups are obtained in some particular cases.