Abelian subgroups of finite p-groups

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Timetable: 10 hours. Class meets on March 2011, Torre Archimede, Room 2BC/30.
The first lecture will be on Monday, March 14 from 14:00 to 16:00, and the following ones on March 15, 16, 22 and 23, from 9:30 to 11:30.

Course requirements: To have followed an undergraduate course on group theory, and to know the most basic theory of finite p-groups.

Examination and grading: Oral examination

SSD: MAT/02 Algebra

Aim: The goal is to introduce the student to an important subject in the theory of finite p-groups, namely, the impact of abelian subgroups on the structure of the group.

Course contents:

Restrictions on the abelian subgroups of a finite p-group G have usually an influence on the structure of G. For example, if all abelian subgroups of G are cyclic, then G is either cyclic or a generalized quaternion group, whereas if all abelian normal subgroups are cyclic, then G is cyclic, dihedral, semidihedral or generalized quaternion. This course is focused on several questions regarding abelian subgroups in finite p-groups. More precisely, we will study:

1. How the rank of the abelian normal subgroups controls the rank of the whole group. For $p > 2$, we will prove an important result of Thompson showing that, if all abelian normal subgroups can be generated by k elements, then the rank of G is at most $k(k+1) = 2$.
2. Whether the existence of abelian subgroups of some particular kind implies the existence of abelian normal subgroups of the same kind. In this direction, we will study important contributions of Alperin and Glauberman, and we will also provide some counterexamples.
3. Some counting theorems for the number of abelian subgroups of some special types. This has also implications in the question of the previous item.