Dynamics in Hamiltonian systems

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Timetable: 20 hrs. Lectures on May/July 2012 (see the calendar), Room 2BC/30, Torre Archimede. **Timetable to be confirmed**.

Course requirements: Basic Lagrangian and Hamiltonian mechanics.

Examination and grading: Seminar on a subject assigned by the Instructors.

SSD: MAT/07

Aim: The aim of this course is to present recent problems concerning the dynamics of quasi-integrable Hamiltonian Systems. Starting from the crisis of classical integrability, established by the Poincaré Theorem on non-existence of prime integrals, we describe the dynamical picture emerging from KAM and Nekhoroshev Theorems, the problem of Arnold diffusion, the Aubry-Mather Theorem for symplectic maps and the weak KAM theory. Special emphasis will be given to applications to specific examples, also through the use of numerical methods.

Course contents:

1. Example of simple dynamical systems, conjugation in dynamical systems, symplectic maps, Poincaré section.
2. Integrability in the classical sense, Hamilton-Jacoby Theory. Crisis of classical integrability: Poincaré Theorem on non-existence of prime integrals.
3. The main Theorems of perturbation theory of Hamiltonian Systems: KAM and Nekhoroshev Theorems
4. Numerical examples: dependence of the structure of the phase space from the perturbation parameter, Arnold web.
5. Aubry-Mather sets in symplectic maps. The Aubry-Mather Theorem. Development of the weak KAM theory: effective Hamiltonian. Dynamical interpretation.
6. Examples of nontrivial dynamics in Hamiltonian Systems: analysis and representation of stable and unstable manifolds. Topological mechanisms of diffusion in phase space. Arnold diffusions. Examples.