Matrices, moments and quadrature with applications

Prof. Gérard Meurant[1]

[1] *Bruyeres-le-Chatel (France)*
Email: gerard.meurant@gmail.com
http://gerard.meurant.pagesperso-orange.fr/

Timetable: 24 hrs. Lectures on January 2012 (see the calendar), Room 2BC/30, Torre Archimede.

Examination: The final exam will be held by the Professor during his stay.

SSD: MAT/08 Numerical Analysis

Aim: The aim of this series of lectures is to describe the mathematical relationships between matrices, moments, orthogonal polynomials, Gauss quadrature rules and the Lanczos and conjugate gradient (CG) algorithms to compute bounds for quadratic forms $I[f] = u^T f(A)v$ where u and v are given vectors, A is a symmetric matrix and f is a smooth function.

Course contents:

- **Orthogonal polynomials and properties of tridiagonal matrices.**
  We will recall the properties of orthogonal polynomials linked to Gauss quadrature and we will also introduce a less classical topic, matrix orthogonal polynomials.
- **The Lanczos and CG algorithms and computation of Jacobi matrices.**
  The Lanczos algorithm will be used to generate the recurrence coefficients of orthogonal polynomials related to estimation of $I[f]$.
- **Gauss quadrature and bounds for bilinear forms $u^T f(A)v$.**
  Gauss quadrature rules are used to obtain bounds for integrals related to $I[f]$. The nodes and weights are related to orthogonal polynomials and Jacobi matrices describing the three-term recurrence. We will also describe extensions to the case of a nonsymmetric matrix A.
- **Bounds for elements of $f(A)$.**
  We will consider the computation of bounds for elements of $f(A)$. The functions f we are interested in are A^{-1}, $\exp(A)$ and $\sqrt{A}$.
- **Estimates of error norms in CG.**
  We will show how Gauss quadrature is used to obtain bounds of the A-norm of the error during CG iterations.
- **Least squares and total least squares.**
  The method of Total Least Squares (TLS) looks for the solution of $(A + E)x = c + r$ where E and r are the smallest perturbations such that $c + r$ is in the range of $A + E$. Computing the solution involves quadratic forms for which we can obtain bounds.
- **Discrete ill–posed problems.**
  We will consider the determination of the Tikhonov regularization parameter for discrete ill-posed problems. We will mainly study generalized cross-validation (GCV) and the L-curve criteria which involve quadratic forms.

References:

G.H. Golub, G. Meurant, *Matrices, moments and quadrature with applications*, Princeton University Press, (2010).