Profinite groups and Profinite completions

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Timetable: 20 hrs (10+10). Lectures of the first part (Prof. Detomi) on March/April and of the second part (Prof. Segal) on April 2012 (see the calendar), Room 2BC/30, Torre Archimede.

Course requirements: Basic knowledge of algebra and group theory.

Examination and grading: Oral exam.

SSD: MAT/02 Algebra

Aim: The course will begin with a quick introduction to profinite groups. The second part of the course will be devoted to some special topics. Suppose G is an infinite group. We consider the family $\mathcal{F}(G)$ of all finite quotient groups of G, and ask (1) what can $\mathcal{F}(G)$ tell us about G, and (2) what can G tell us about $\mathcal{F}(G)$? If we assume that G is finitely generated, then knowing $\mathcal{F}(G)$ is equivalent to knowing the *profinite completion* $\widehat{G}$ of G, and question (1) comes down to: what properties of a group are preserved by the functor $G \mapsto \widehat{G}$? These are called 'profinite properties'.

Course contents:
First part (E. Detomi): Topological groups; Inverse limits of groups and profinite groups; Profinite completions of groups; Finitely generated groups.
Second part (D. Segal):
(i) Metamathematical motivation, coming from the area of *decision problems*.
(ii) *Conjugacy separability*: when conjugacy is a profinite property. Examples where it isn't; the case of polycyclic groups, where it is.
(iii) Isomorphism: the $\mathcal{C}$*-genus* of a group G is the set of isomorphism classes of groups $H \in \mathcal{C}$ with $\widehat{H} \cong \widehat{G}$. In general the genus is infinite; when $\mathcal{C}$ is the class of polycyclic groups the genus is finite. We will outline the main ideas in the proof, without full details. This includes some discussion of algebraic groups and arithmetical finiteness theorems.

One version of Question (2) asks: *which profinite groups can be the profinite completions of finitely generated abstract groups*? We will discuss some necessary conditions, related to theorems of Mal'cev and Lubotzky on linear groups. Finally we will outline some sufficient conditions: (a) in the context of Cartesian products of finite groups and (b) in the context of infinitely iterated wreath products, using branch groups.