

# The behavioral approach to control of distributed systems

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**Timetable:** 20 hrs. Class meets every Tuesday and Friday. First lecture on Tuesday, September 18, 2012. Dept. of Information Engineering. **Timetable to be confirmed.**

**Course requirements:** The basic of Kalman's state space theory, linear algebra Standard linear algebra and probability theory.

**Examination and grading:** Homework and final examination.

**Aim:** To introduce the key ideas of the behavioral approach to control of open dynamical systems, and to show how the ideas of J.C.Willems, developed first as a generalization of the Kalman theory to the case of lumped systems, carry over naturally to distributed systems. Just as linear algebra is the language in which the Kalman theory of state space systems is written, the behavioral theory of distributed systems is written in the language of commutative algebra. The course will not assume any background, but will develop the required results from commutative and homological algebra along the way.

**Course contents:**

**Lecture 1.** A quick review of controllability in the Kalman theory of state space systems; how do we generalize this theory to the case when first order operators are replaced by operators of arbitrary order and when we ignore input-output structures.

**Lecture 2.** A little commutative algebra (commutative rings, modules, localization, Hom and tensor product).

**Lecture 3.** The generalization to distributed systems (described by constant coefficient partial differential operators); controllability as a patching problem, controllability versus potential.

**Lecture 4.** Necessary and sufficient conditions for controllability of C1 behaviors; controllable and autonomous behaviors.

**Lecture 5.** A little more commutative and homological algebra (injective and at modules); The Fundamental Principle of Malgrange-Palamodov (statement).

**Lecture 6.** Consequences of the Fundamental Principle for C1 behaviors - elimination, lattice structure etc.

**Lecture 7.** Behaviors in other function spaces such as the space of compactly supported smooth functions; the Nullstellensatz problem for systems of partial differential equations, the problem of calculating Willems closures.

**Lecture 8.** A last bit of commutative algebra (associated primes, primary decomposition); the Nullstellensatz for systems of PDE.

**Lecture 9.** Other structures on behaviors - causality, interconnections, feedback, stability.

**Lecture 10.** Further research directions.

**References:**

There is no textbook in the subject (yet) but I shall pronotes. Apart from papers in the subject, there is a comprehensive survey: J.C.Willems: *The behavioral approach to open and interconnected systems*, IEEE Control Syst. Mag., 27:46-99, 2007.