

Numerical methods for Ordinary Differential Equations

Prof. Marino Zennaro¹, Prof. Rossana Vermiglio²

¹University of Trieste, Mathematics and Computer Science
Email: zennaro@units.it

²University of Udine, Department of Mathematics and Computer Science
Email: Rossana.Vermiglio@uniud.it

Timetable: 16 hrs (Part I, Prof. Zennaro), January (see the calendar, Rooms 2AB/45 and 2BC/30) + 12 hours (Part II, Prof. Vermiglio), March (see the calendar, Rooms 2BC/30 and 2BC/60). Torre Archimede.

Course requirements: it is advisable to have attended a basic course in Numerical Analysis.

Examination and grading: A unique written exam for both Part I and Part II.

SSD: MAT/08 Numerical Analysis

Aim: We present basic numerical methods for initial value problems in ordinary differential equations and we analyse their convergence and stability properties.

Course contents:

Part I

Existence and uniqueness of the solution and continuous dependence on the data for the initial value problem $y'(x) = f(x, y(x))$, $y(x_0) = y_0$.

Classical Lipschitz constant and right hand side Lipschitz constant.

General one-step methods; explicit and implicit Runge-Kutta methods.

Definition of local truncation and discretization error for one-step methods and definition of consistency of order p .

Convergence theorem with order p for one-step methods. Order conditions for Runge-Kutta methods. Order barriers for explicit and implicit methods.

Variable stepsize implementation. Embedded pairs of methods of Runge-Kutta-Fehlberg and Dormand-Prince type.

Part II

Introduction to the stability of numerical methods. Stiff problems.

Definition of A-stability, AN-stability and BN-stability of a numerical method.

Analysis of A-stability for Runge-Kutta methods: A-stability regions. L-stability.

Analysis of AN-stability and BN-stability for Runge-Kutta methods. Algebraic stability.

The phenomenon of the order reduction: an example. B-convergence.

Short analysis of A-stability for linear multistep methods. $A(\alpha)$ -stability and stiff-stability. Backward differentiation formulas.

References:

- E. Hairer, S.P. Norsett, G. Wanner: Solving Ordinary Differential Equations I, Nonstiff Problems, Springer-Verlag, Berlin, 1993
- E. Hairer, G. Wanner: Solving Ordinary Differential Equations II, Stiff Problems, Springer-Verlag, Berlin, 1993
- J.C. Butcher: Numerical methods for ordinary differential equations. Second edition, John Wiley & Sons, Ltd., Chichester, 2008
- J.D. Lambert: Numerical methods for ordinary differential systems. John Wiley & Sons, Ltd., Chichester, 1991
- Lecture notes by the professors