Introduction to Leavitt path algebras

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Timetable: 12 hrs. First Lecture on May 23, 10:00-13:00, for the other lectures see the calendar. Torre Archimede, Room 2BC/30.

Course requirements: Only a basic knowledge of ring theory (perhaps at the level of Hunger-Ford's "Algebra" book from the Springer Graduate Texts in Mathematics series) is required as prerequisite for this course.

Examination and grading: Students will be required to complete a set of exercises about Leavitt path algebras in order to pass the course.

SSD: MAT/02 Algebra

Course contents:

Leavitt path algebras are a class of algebras (defined over any field K) which were introduced in 2004. They have as their motivation a class of C_*-algebras, but their study has taken on a life of its own. They arise in a natural way from directed graphs. Many basic examples of algebras arise as Leavitt path algebras (e.g., matrix rings and Laurent polynomial rings). But many additional quite interesting algebras arise in this context as well, including the classical algebras $LK(1, n)$ (for any integer $n \geq 2$) studied by W.G. Leavitt in the early 1960's. Leavitt path algebras are very appealing because a great deal of ring-theoretic information about them can be immediately read from the corresponding directed graph. In the first half of this course we will learn the definition of Leavitt path algebras, and will study some of the basic examples. The pace will be leisurely enough for all students to understand the underlying ideas. We will discuss the original motivation for constructing these algebras. Then we will move to the "early" results. These are results of the form: The directed graph E has graph-theoretic property P if and only if the corresponding Leavitt path algebra $LK(E)$ has ring-theoretic property Q. For example, we find those 1graph properties P which correspond to the ring-theoretic properties $Q =$ simple; finite dimensional; prime; and others. In the second half of the course we will do a number of things. First, we will introduce and review some important, standard ring-theoretic concepts, such as purely infinite simplicity; exchange; primitivity; von Neumann regularity; stable rank; and the (non-associative) bracket Lie algebra of an associative algebra. For each of these ring-theoretic properties we will also give a *The directed graph E has graph-theoretic property P if and only if the corresponding Leavitt path algebra $LK(E)$ has ring-theoretic property Q* result, sometimes with full proof, sometimes simply with motivation. But more importantly, we will show how Leavitt path algebras have been used as tools to answer significant, sometimes-long-standing ring-theoretic questions in the context of each of these properties. Second, we will show how Leavitt path algebras and a class of C_*-algebras called "graph C_*-algebras" are very closely related. We will give some proper- ties of these graph C_*-algebras, and show various similarities and differences between the two theories. So the course will be very informative for students who study operator algebras. Finally, we will motivate some of the open problems in Leavitt path algebras. The most important of these problems is the Kirchberg Phillips Question for Leavitt path algebras. We will give a status of the problem, and show how it is related to various questions in symbolic dynamics.