Vector bundles, principal bundles and connections

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Timetable: 16 hrs. First lecture on March 22, 2013, 11:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic knowledge of differential geometry and Lie group theory.

Examination and grading: Oral exam.

SSD: MAT/03 Geometry.

Aim: Vector bundles and, more generally, principal bundles and connections on bundles play a very important role in modern differential geometry. They also have important applications in algebraic geometry, topology and theoretical physics, where they form part of the foundational framework of gauge theories. The aim of this course is to give an introduction to the study of vector bundles and principal bundles, and of connections defined on them. Being an introductory course, we shall try to keep the necessary prerequisites to a minimum: a basic knowledge of differential geometry and Lie group theory will help, but is not strictly necessary. At the end of the course, if time permits, we will discuss some applications of principal bundles to physical theories, like Maxwell equations (electromagnetism) and Yang-Mills equations.

Course contents:

- Vector bundles
- Connections on vector bundles
- Flat bundles and flat connections
- Hermitian bundles and hermitian connections
- Chern classes
- Principal bundles
- Fibre bundles associated to a principal bundle
- Connections on principal bundles
- Holonomy groups
- Connection and curvature forms
- Flat connections
- Some applications (if time permits): Maxwell equations and Yang-Mills equations