D-modules Theory

Prof. Bruno Chiarellotto and Dr. Giovanni Morando[1]

[1] University of Padova
Department of Mathematics
Email: {chiarbru,gmorando}@math.unipd.it

Timetable: 20 hrs. First lecture on January 21, 2013, 10:30-12:30, Room 2BC/30, Torre Archimede. The first 14 hrs of the lectures have been fixed (see the calendar). The others hrs will be fixed according with the attending students.

Course requirements: Analysis, geometry and algebra at undergraduate level and basics in homological algebra and sheaf theory

Examination and grading: starting from the material presented during the class, we will ask the Ph-D student to attack a precise development of the subject of the D-modules which we will tailor on the basis of the his main math interest.

SSD: MAT/03 Geometry

Course contents:
The theory of D-modules was born in the end of the '60's simultaneously in the work of J. Bernstein in Russia and M. Sato in Japan. It consists in an algebraic and functorial study of linear systems of partial differential equations on complex manifolds. The initial motivations and the techniques developed in the first stages witness the ample range of applications and tools involved in this field of research. Indeed, Bernestein's initial motivation was the results about divisions of distributions while the Japanese school was interested in microlocal analysis. The techniques developed at the very beginning were related to the algebraic properties of the ring of linear differential operators with holomorphic coefficients, D, and they made use of homological methods. Soon after, index theory, algebraic geometry, theoretical physics, singularity theory and representation theory interlaced D-modules theory in very fruitful and unexpected ways. Nowadays such a topic is still a very active field of research and it is a source of inspiration in many branch of mathematics. The course aim to give an overview of the algebraic and geometric techniques involved in D-modules theory and to explain important analytic results based on topological properties of analytic spaces. The course will cover the use of some algebraic tools related to D-modules (as filtrations and homological algebra), geometric objects and constructions (as the characteristic variety and the six Grothendieck operations) and invariants of linear PDEs (as irregularity) particularly meaningful in the analytic study of PDEs. Emphasis will be given to the cases of Riemann surfaces or local settings where the situations are somehow simpler still providing examples of general and interesting phenomena. The final exam will consist in explaining and detailing some topics mentioned during the course either classical or recent following the interests of the students as hyperfunctions, asymptotic analysis, sheaf cohomology and microfunctions, Riemann-Hilbert correspondence, Hodge theory, derived categories, perverse sheaves and representations of quivers.