Representations and weak convergence methods for the analysis and approximation of rare events

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Timetable: 24 hrs, First lecture on May 8, 2013, 11:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: Standard knowledge of Probability and measure theory. Background material will be covered by Markus Fischer and/or Paolo Dai Pra in some introductory lectures.

Examination and grading: Seminar on a subject assigned by the Instructors

SSD: MAT/06 Probability and Statistics

Aim: The course will illustrate the role played by *rare events* in problems arising in analysis, probability and statistics. Examples of such problems are: PDE with small noise, homogenization problems, mean field interactions, discontinuous statistics. Numerical Monte Carlo schemes will also be illustrated.

Course contents: Elements of the following could be covered as background material or be part of the course:

1. Basics of large deviation theory and weak convergence
2. Properties of relative entropy
3. Hamilton-Jacobi-Bellman (HJB) equations, their relationship to control, and verification arguments
4. Basics of Monte Carlo approximation and importance sampling

The outline of the course would be as follows, though it would be modified depending on what is covered as preliminary material.

1. Preliminaries and general results in large deviation theory and relative entropy Some of the general results in large deviation theory (such as the equivalence between a large deviation principle and the Laplace principle), as well as important properties of relative entropy.
2. Elementary examples Here we state (without proofs) representations for one or more simple discrete time examples (e.g., product measure for Sanovos theorem) and a simple continuous time case (e.g., Brownian motion for small noise diffusions), and then show how weak convergence methods can be used to prove various large deviation estimates.
3. Discrete time processes We start by deriving representations for a class of discrete time models using the chain rule for relative entropy. We then do selected applications such as (i) the empirical measure for a Markov process and (ii) small noise stochastic difference

equations. The examples will be chosen so they can also be used when discussing Monte Carlo approximation.

4. Representations for continuous time We present representations for processes in continuous time, including functionals of infinite dimensional Brownian motion and Poisson random measures, and sketch the proofs.
5. Continuous time processes We show how the representations may be used for the large deviation analysis of problems in continuous time. The particular problems treated may include (depending on preferences) small noise stochastic partial differential equations, averaging and/or homogenization problems, mean-field interactions, a discontinuous statistics problem, etc. As in the case of discrete time, we will set up some of the Monte Carlo examples.
6. Monte Carlo approximation Here we will focus on how one can use of the same ideas (representations and weak convergence) for the design and analysis of Monte Carlo schemes, emphasizing importance sampling. Topics will include the following.
   a. Generalities on Monte Carlo and importance sampling, rare event issues, open loop versus dynamic changes of measure, and the role of a differential game.
   b. Simple example problems (e.g., hitting probabilities, finite time problems) and the associated HJB equations.
   c. Subsolutions and their use in the design of importance sampling schemes.
   d. Verification arguments to assess performance.
   e. Examples of subsolutions.
   f. Higher moments.
   g. Other types of Monte Carlo schemes.