Stochastic Volatility Models

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Timetable: 20 hrs, September 2013, Torre Archimede, Room 2BC/30.

Course requirements: Probability with measure theory. Standard knowledge of Stochastic Analysis.

Examination and grading: Seminar on a subject assigned by the Instructors

SSD: MAT/06 Probability and Statistics.

Aim: The course will illustrate some recent methodologies in Mathematical Finance.

Course contents: This series of lectures is intended to provide a good knowledge of some recent methodologies used in Mathematical Finance which aim at going beyond the shortcomings of the celebrated model proposed in 73 by Black & Scholes, who assumed that the volatility of the asset price is constant. The fact that a constant volatility is empirically rejected opened the door to a wide class of approaches that tried to reconcile the risk-neutral pricing methodology with the real market prices. The most famous model is the one proposed by Heston (1993), who assumed a dedicated stochastic differential equation for the volatility process. This model is able to reproduce most stylized facts that are observed in the option market, like the volatility smile and the skew effect. However, the asset price is no more lognormal as in the Black & Scholes framework and the pricing procedure requires different techniques, mostly based on the Fast Fourier Transform (FFT). Nevertheless, the pricing can be performed essentially in closed form (modulo the computation of a numerical integral), so that the Heston model can be considered to be tractable. The aim of this course is to provide a systematic investigation of the stochastic volatility models by using a unified approach based on the FFT technology. The support for the lectures consists in some notes provided by the teacher and papers that are available on the web.

1. In the first part of the course we review the risk neutral pricing methodology, the B & S model and we refresh the hedging problem in a constant volatility setting. We introduce the notion of implied volatility surface and the volatility smile and skew
2. In lecture 2 we introduce the stochastic volatility model of Heston together with the FFT methodology. In particular we decompose the price as an integral involving the product of the Fourier transform of the asset price (which is model dependent) times the Fourier transform of the payoff (model independent but dependent on the particular derivative we are pricing).
3. In lecture 3 we compute the characteristic function of the asset price in the Heston model. This problem can be solved in different ways, e.g. by solving a linear 2 order ODE or by linearizing the corresponding Riccati ODE and solving the associated system of 2 linear

ODEs. We proceed in both ways since it is useful in order to get some intuition for the Wishart matrix case.

4. In lecture 4 we extend the procedure to the case where the volatility of the asset is multi-factor and we discuss the solvability of the corresponding Riccati ODE. We also show an approach in the Forex market where we will calibrate simultaneously 3 implied volatility surfaces in a triangle of currencies.
5. In lecture 5 we introduce the notion of Affine class of stochastic processes in the classic state space domain. We discuss the role of the correlation structure in the solvability of the characteristic function. We also discuss the presence of jumps in the dynamics of the underlying as well as in the dynamics of the volatility process.
6. In lecture 6 we extend the notion of affine class to the case where the state space domain is the set of positive semidefinite matrices. In particular we introduce the Wishart process, which represents the natural extension of the square root process (squared Bessel process) to the multivariate setting.
7. In Lecture 7 we investigate some basic properties of the Wishart process, like the affinity of its infinitesimal generator and the exponentially affine form of its characteristic function. We develop the computation by using the linearization of the matrix Riccati ODE in fully analogy with the scalar case.
8. In lecture 8 we apply the matrix technique to a stochastic volatility model where the volatility factors are non trivially correlated: this goes beyond the classic multi-Heston framework where factors have to be independent in order to grant analyticity.
9. In lecture 9 we introduce a multi-asset volatility model where the Wishart process represents the instantaneous variance-covariance matrix among the assets. We show that the model is fully tractable and it can be calibrated to the market.
10. In lecture 10 we give some series expansions of prices and implied volatilities under the previous models. The expansion are in terms of the volatility of volatility and with respect to the time to maturity of the option. This is useful in view of calibrating the models (they provide some proxies that can be used as starting point for the optimization algorithm).