Topics on optimal control and PDEs

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Timetable: 20 hrs (period and dates to be defined), Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/05 Mathematical Analysis

Aim: The course deals with the analysis of optimal control problems and of the related first order PDEs of dynamic programming. In particular, we shall focus our attention on time optimal control problems for linear and non- linear systems. We shall present some recent results concerning the regularity and the compactness of viscosity solutions to Hamilton-Jacobi and Hamilton-Jacobi-Bellmann Equations. The course will cover topics in nonsmooth analysis, geometric measure theory, optimal control and viscosity solutions.

Course contents:

1. A model problem
   - A problem in calculus of variations + Dynamic programming principle
   - The Hopf formula + Hamilton-Jacobi equation, viscosity solutions
   - The method of characteristics
2. Time optimal control for linear systems + Controllability
   - Bang-bang principle
   - The continuity of the minimum time function
3. Regularity of minimum time for nonlinear systems
   - Semiconcave function
   - Fine properties: differentiability and rectifiability
   - Differentiability for a class of non-Lipschitz functions
   - Application to minimum time
4. Compactness properties of solutions to Hamilton-Jacobi equations
   - The semiconcavity of solutions of Hamilton-Jacobi equations
   - A controllability result
   - The kolmogorov ε-entropy estimate for a class of semiconcave functions.
   - A connection with scalar conservation laws.