Recent advances in Finance and Stochastics

Nicole El Karoui & Monique Jeanblanc

Univ. Pierre et Marie Curie & Univ. d'Évry Val d'Essonne)

Lecture schedule: 18h, in May 2014, room 1BC45[1], unless explicitly mentioned.

PART A) May 22 ($15-18$, room 2BC30), May 26 ($10-13$), May 27 ($10-13$);

PART B) May 26 ($14.30-17.30$), May 27 ($14.30-17.30$), May 28 ($10-13$).

Course requirements: Probability and Stochastic Calculus.
Examination: written.
SSD: MAT/06.

Contents

PART A) Enlargement of filtrations

The course is devoted to enlargement of filtration with Finance in view. Two filtrations $\mathbb{F}$ and $\mathbb{G}$ are given, such that $\mathbb{F} \subset \mathbb{G}$. The goal is to study the behavior of $\mathbb{F}$-martingales and to give conditions so that these martingales are $\mathbb{G}$ semi-martingales. In that case, the decomposition of any $\mathbb{F}$-martingale as a $\mathbb{G}$ semimartingale will be provided. The two main examples of such filtrations $\mathbb{G}$ are the case of initial enlargement, where $\mathbb{G} = \mathbb{F} \vee \sigma(L)$, where L is a random variable, and the case of progressive enlargement, where $\mathbb{G}$ is the smallest filtration which contains $\mathbb{F}$ and makes a given positive random variable τ a stopping time. The case where $\mathbb{F}$ is a Brownian filtration will be studied with more details. Applications to arbitrage opportunities in finance will be given.

We will provide answers to the above questions and see some recent applications of this theory to Finance.

Lecture I

The basic model of credit risk modelling involves a specific case of progressive enlargement of filtration, namely the case where any $\mathbb{F}$-martingale is a $\mathbb{G}$-martingale. If the filtration $\mathbb{F}$ enjoys a predictable representation theorem (for example, if $\mathbb{F}$ is the filtration generated by a Brownian motion), then a predictable representation theorem is valid in the enlarged filtration. The case where there are discontinuous $\mathbb{F}$-martingales is more involved, and we shall study some examples.

Lecture II

[1]Dipartimento di Matematica, Torre Archimede, Via Trieste 63, 35121 Padova.

If τ is a random time with a positive conditional density, i.e. if $\mathbb{P}(\tau > u|\mathcal{F}_t) = \int_u^\infty p_t(\theta)f(\theta)d\theta$ for some $\mathbb{F}$ adapted positive process $p(\theta)$, where f is the density of τ, it is easy to prove that any $\mathbb{F}$-martingale is a semi-martingale for the initially enlarged filtration $\mathbb{G} = \mathbb{F} \vee \sigma(\tau)$ and for the progressive enlarged filtration. In that case, there are no arbitrages opportunities induced by the enlargement, however, the more informed agent makes a profit, characterized by the drift information (in the logarithm case).

Lecture III

In a general case, any $\mathbb{F}$-martingale stopped at τ is a $\mathbb{G}$-semi martingale, with an explicit decocmposition. For a specific case of random times τ (called honest times), and in a complete financial market, there are classical arbitrages before τ which can be given in an explicit form. For the general case of random times, under some weak condition, there are no arbitrages of the first kind, meaning more or less that there exists a positive $\mathbb{G}$-local martingale L so that $S^\tau L$ is a $\mathbb{G}$-local martingale , where S^τ is the stopped process associated with the price of the assets in an $\mathbb{F}$-arbitrage free financial market.

The case after τ is more difficult, and we shall pay attention only to honest times. In this last lecture, we shall restrict our attention to the case where the filtration $\mathbb{F}$ will be a Brownian filtration, even if the results can be extended to any filtration.

References

[1] Aksamit, A., Choulli, T., Deng, J., and Jeanblanc, M.: *Non-Arbitrage under Random Horizon for Semimartingale Models*, Preprint of University of Alberta and Evry-Val d'Essonne University, 2014.

[2] Aksamit, A., Choulli, T., Deng, J., and Jeanblanc, M.: *Arbitrages in a progressive enlargement setting*, Arbitrage, Credit and Informational Risks, Peking University Series in Mathematics, Vol. 6, 55-88, World Scientific (2014)

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PART B) Microsimulation and population dynamics

Most countries are experiencing a reduction in mortality over time, which is a new phenomenon, without any historical reference. In this context, societies are facing new challenges, in particular concerning generation equilibrium, the role of ageing populations and the viability of shared collective systems, in particular pension systems. In so-called structured populations, individuals differ according to variables that influence their survival and/or reproduction abilities. These variables are from different types, depending of the specificities of the population in consideration. For instance, in human population, key characters are gender, socioeconomic level, spacial location, marital status and age. For example, the "population" of firms interacting in the counterparty risk of a bank may be characterized by sector, countries, rating, "age" and other specificities. In the study of the order book in HFT by Hawkes process, the main factor is the age of the order. We are concerned with the dynamics of populations with trait and age structures.

Lecture I - Introduction and description of the classical methodology

Introduction to basic life insurance and to some mortality models for annual death probabilities. Example: Life insurance products, called "Variable Annuities".

Introduction to Marked Point processes, and Birth and Death processes.

Lecture III - Individual-based model and population dynamics

Introduction of a dynamic population model, inspired by [5] and [4] in the field of ecology, and the work of [1] for demographic purposes, that allows to take into account individual characteristics (called "traits" as gender, socio-economic characteristics...) which have a real impact on demographic behavior and age. Example: a new simulation method of Hawkes processes used in credit modeling or HFTrading.

Lecture III - Applications to longevity risk dynamics using empirical data Back to longevity risk modelisation, utilization of the population dynamics model to simulate some stylized fact as the cohort effect, and more generally the importance of the composition of the underlying population.

References

[1] H. Bensusan. *Risques de taux et de longévité: Modélisation dynamique et applications aux produits dérivés et à l'assurance vie*. PhD thesis, École Polytechnique, 2010.

[2] A. J. G. Cairns, D. Blake and K. Dowd. *Monte Carlo Algorithms for Default Timing Problems*. Journal of Risk and Insurance, 73: 687–718.

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