Lie Algebras

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Timetable: 25 hours. Lectures on Wednesday 11:30-13:15 and Thursday 09:30-11:15. First lecture on April 15, 2015, Torre Archimede, Room 2AB/40.

Course requirements: Basic notions of linear algebra

Examination and grading: exercises

SSD: MAT/02

Aim: This course provides an introduction to Lie algebras and aims at presenting the classification of complex simple Lie algebras.

Course contents:

1. Basic notions. The adjoint representation and its subrepresentations. Derived subalgebra. Solvable and nilpotent Lie algebras. Nilpotent elements are ad-nilpotent.
2. Engel's theorem and Lie's theorem.
3. Irreducible representations of solvable Lie algebras. Schur's lemma.
4. Irreducible representations of sl(2,C). Uniqueness of the Jordan decomposition in End(V)
5. Killing form. Cartan's solvability criterion.
6. Cartan's semisimplicity criterion. Trace forms and Casimir element. Weyl's theorem.
7. Cartan subalgebras. Abstract Jordan decomposition.
8. The root space decomposition. sl2-triples.
9. Reductive Lie algebras. Root strings. Euclidean structure on the real span of roots.
10. Root systems and Weyl group.
11. Strategy for the classification of classical Lie algebras. Simple Lie algebras have irreducible root systems and viceversa.
12. Classical Lie algebras are simple (up to two cases).
13. Serre's theorem. Uniqueness of the semisimple Lie algebra associated with a root system. Uniqueness of the root system associated with a Lie algebra.