Symmetry, Lie groups and dynamical systems

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Timetable: 24 hrs. First lecture on September 3, 15:00, 2015, (later dates to be confirmed) Torre Archimede, Room 2BC/30.

Course requirements: Basic notions of differential geometry (calculus on manifolds, differential forms). Some basic notions of differential equations and of Hamiltonian mechanics (canonical transformations, integrability, elementary examples) are desirable but not strictly necessary.

Examination and grading: Oral exam, on the content of the course and/or supplementary material (to be decided case by case).

SSD: MAT/07

Aim: The presence of symmetry plays an important role in the study of differential equations and mechanical systems; in particular, symmetry is a key ingredient of integrability. Mathematically, a (continuous) symmetry is formalized as invariance of a vector field under a smooth action of a Lie group and allows its reduction to the quotient manifold; then, reconstruction techniques aim at obtaining information on the complete system from the reduced one. Integrability is the result of the combined presence of symmetry and first integrals. A special case is Hamiltonian systems, where symmetry groups produce first integrals as well (Noether theorem, momentum map). The course provides an introduction to these topics.

Course contents:
Basic examples of Lie groups ($SO(3)$, S^3, quaternions, etc). Differential geometry of Lie groups (left invariant vector fields, Lie algebra of the group, exponential map, etc). Smooth action of a Lie group on a manifold; quotient manifold. Reduction of equivariant ODEs. The mechanical case: the momentum map; symplectic reduction. Application: actions of tori and integrability of ODEs. Reconstruction of the reduced dynamics: relative equilibria; maximal tori and relative periodic orbits.

References:

- J.M. Lee, *Introduction to smooth manifolds. Graduate Texts in Mathematics*, 218. Springer, New York, 2003 (1st edition), 2013 (2nd edition).

- J.E. Marsden and T.S. Ratiu, *Introduction to mechanics and symmetry. Texts in Applied Mathematics*, 17. Springer-Verlag, New York, 1994.

- P. Libermann and C-M. Marle, *Symplectic geometry and analytical mechanics.* Mathematics and its Applications, 35. D. Reidel Publishing Co., Dordrecht, 1987.