Statistical methods

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Timetable: 24 hrs. (two lectures of two hours each, per week). Class meets every Monday and Wednesday from 2:30 to 4:30, starting on Monday, April 20-th, 2015. Meeting Room DEI/G (3-rd floor, Dept. of Information Engineering, via Gradenigo Building).

Course requirements: Basics of Probability Theory and Linear Algebra.

Examination and grading: homework assignments and take-home exam.

Aim: The course will present a survey of statistical techniques which are important in applications. The unifying power of the information theoretic point of view will be stressed.

Course contents:

Background material. The noiseless source coding theorem will be quickly reviewed in order to introduce the basic notions of entropy of a probability measure and informational divergence (Kullback- Leibler distance) between two probability measures.

Divergence minimization problems. Three divergence minimization problems will be posed and, on simple examples, they will be connected with basic methods of statistical inference: ML (maximum likelihood), ME (maximum entropy), and EM (expectation-maximization). We will solve some instances of the ME problem to show the interplay between linear and exponential families of probability measures.

Multivariate analysis methods. Linear regression analysis: OLS ordinary least squares and related methods: GLS (generalized least squares), TLS (total least squares). Connection with ML under Gaussian assumptions and the Gauss Markov theorem. PCA (principal component analysis) for empirical data and random vectors: position of the problem, derivation of the solution, geometrical interpretation. Applications of PCA to least squares: PCR (principal component regression) and PLS (partial least squares). Approximate matrix factorization and PCA, with a brief detour on the approximate Nonnegative Matrix Factorization (NMF) problem. Canonical Correlations (CC): position of the problem and derivation of the solution. Factor Analysis: position of the problem. For most of these methods there is a natural interpretation in terms of divergence minimization which will be reviewed.

EM methods. The Expectation-Maximization method will be introduced as an algorithm for the computation of the Maximum Likelihood (ML) estimator with partial observations (incomplete data) and interpreted as an alternating divergence minimization algorithm *à* la Csiszár Tusnády.

Hidden Markov models. We will introduce the simple yet powerful class of HMM (hidden Markov models) and discuss parameter estimation for HMMs via the EM method.

References: A set of lecture notes and a list of references will be posted on the web site of the course.