From sparse optimization to sparse optimal control

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Timetable: 10 hrs. First lecture on June 8, 2015, 11:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: The mathematical tools: linear algebra, basic convex optimization, theory and numerics of ODE, probability measures, mean-field limits, Gamma-convergence

Examination and grading:

SSD: MAT/05

Aim: At the end of this course the students will have learned the fundamental concepts of sparse optimization. They will have revised the theory of optimal control of ODEs and its specification for sparse control and sparse controllability. They will learn the techniques associated to mean-field limits of ODEs systems and the Gamma-convergence of optimization problems.

Course contents:
This course will start with an introduction to sparse optimization with linear constraints. We dedicate the first lecture to the exploration of the world of compressed sensing and its several algorithmic approaches. The second lecture will be addressed to generalizing the framework to nonlinear constraints. We present two different kind of nonlinearities, the first is of quasi-linear type, the second will be of low-rank type. Having generalized sparse optimization to nonlinear constraints allows us to look at sparse optimal control problems with ODE constraints as a subclass of the more general framework of nonlinear sparse optimizations. The subject of the third lecture will be the concept of sparse optimal control of ODE systems modeling social interactions, with a more precise quantitative analysis of consensus models. The forth lecture will be dedicated to the mean-field limits of sparse optimal control problems. We introduce a new class of finite dimensional optimal control problems with ODE constraints which admit an infinite dimensional counterpart with PDE constraints and retain a certain sparsity of the control. The fifth and last lecture will be dedicated to mixed diffuse-granular modeling were sparse optimal control problems will be constrained by coupled systems of ODEs and one PDE.