Minimal Discrete Energy Problems

Prof. Edward B. Saff[1]

[1] Vanderbilt University, Nashville, TN, USA
Department of Mathematics
Email: edward.b.saff@vanderbilt.edu

Timetable: 20 hrs. First lecture on June 23, 2015, 11:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: Numerical methods for ordinary differential equations.

Examination and grading:

SSD:

Aim:

Course contents:

The course will focus on the general topic of finding (computing) and analyzing configurations of points that are optimally or near-optimally distributed on a set. Such questions arise in a number of guises including best-packing problems, coding theory, geometrical modeling, statistical sampling, radial basis approximation, self-assembling materials, and even golf-ball design (i.e., where to put the indentations). Special emphasis will be given to the behavior (for large N) of N-point equilibrium configurations on a compact set A for the Riesz potential $(1/r)^s$, where $s > 0$ is a parameter and r denotes Euclidean distance between points. [The case $s = 1$ in R^3 corresponds to the familiar Coulomb potential, while large s corresponds (in the limit) to best-packing.] The analysis of such points falls under the umbrella of classical potential theory when $s < d = \dim(A)$ and is a consequence of the continuous theory. But what if $s > d$ or $s = d$? In such cases, the classical theory does not apply and new techniques are needed to analyze the behavior of minimal energy configurations. We shall describe these techniques and related low-complexity techniques for computations.