Ordered K-theory
(The theory of additive and non-negative invariants for rings and modules.)

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Timetable: 16 hrs. First lecture on March 9, 2015, 14:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basic notions on module categories and homological algebra.

Examination and grading:

SSD: MAT/

Course contents:

Ultimately we only understand our objects in terms of invariants, usually real or integer valued: Cardinality, Length, Volume, Area, Probability, Weight, Dimension, Rank, State, Euler characteristic, Multiplicity, Entropy, etc
All these are non-negative and have an additive property: the value of the whole is the sum of the values of its parts or sub and factor. The generic object for these functions is the object of study of these lectures: the K-group (Grothendieck group) equipped with a natural partial order.

Tentative plan of topics:

1. The ordered K-group (KO) of a category. Examples. Length functions. Functorial properties. Extensions by devissage and by resolutions. The KO-series and KO-dimension of a Serre category.
2. Computation of the KO group (series, dimension) of special categories: Noetherian modules (more generally modules with Krull dimension). Projective modules the Goodearl-Warfield Theorem. Modules with finite free resolution. Euler characteristic. Macraes invariant. Valuation rings.
3. Links with functional analysis: Elliotts theorem (KO is a complete invariant for certain C* algebras). von Neumann regular rings
4. Applications: entropy, multiplicity, state. Rank rings. Zero divisor problem for group rings.

There are lots opportunities for further research and open problems, these will be stated as we go along.