The fundamental Group in its different realizations

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Timetable: 24 hrs. First lecture on October 13, 2015, 11:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic notion of topology, geometry and algebra.

Examination and grading: The exam will be taylored on the basis of the students which will attend this introductory course.

SSD: MAT/02 - MAT/03

Aim: to show how the notion of fundamental group can have different realization i.e. it can be built in different settings, not only in topology.

Course contents:
Starting from the topological definition using paths (and connecting it with singular homology theory), we will also introduce its definitions using coverings transformations. We then study the representations of the fundamental group. Using the riemann-hilbert correspondance we will see how such a representations are linked to connections. After that we will try to introduce the analogous definitions in arithmentic/algebraic terms. What should be the analogous of ”covering” in the algebraic setting? What should be the analogous of representations of the fundamental group? This will lead to the definitions of ”etale” coverings (In particular the fundamental group of a field as its galois group), to the theory of tannakian categories (those ones which are equivalent to some categories of representations). We then try to use the fundamental group as an invariant of the space: this will lead us to the theory of motives.