Convex polytopes

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Timetable: 12 hrs. First lecture on January 18, 2016, 14:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic knowledge of the theory of linear programming and polyhedra: this can be achieved by reading Chapters 7 and 8 in [1] or Chapter 3 in [2] before the beginning of the course.

Examination and grading: Written exam.

SSD: MAT/09.

Aim: Polytopes arise in optimization, but have been studied for long time, e.g., in Physics, Chemistry and Biology. We survey some classical and very recent results on the structure of polytopes.

Course contents:

1. Definition and examples of polytopes: 3-dimensional polytopes (including Platonic solids), cyclic polytopes, permutahedron, combinatorial polytopes.
2. Representations of polyhedra: $\mathcal{H}$-polyhedra and $\mathcal{V}$-polyhedra.
3. Affine hull, recession cone, lineality space.
4. Facets, vertices and extreme rays.
5. Face lattice of a polytope.
6. Skeleton of a polytope and Balinski's theorem.
7. Diameter of a polytope and the Hirsch conjecture: sub-exponential upper-bounds, correctness of the Hirsch conjecture for 0/1 polytopes, counterexample to the Hirsch conjecture (sketch), polynomial upper bound for totally unimodular matrices.

References:

[1] A. Schrijver, *Theory of Linear and Integer Programming*, Wiley, 1986.

[2] M. Conforti, G. Cornuéjols, G. Zambelli, *Integer Programming*, Springer, 2014.