Random Graphs and Stochastic Geometry in Networks

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Timetable: 20 hrs. Class meets every Monday and Wednesday from 10:30 to 12:30. First lecture on Wednesday, March 30th, 2016. Room DEI/G (3-rd floor, Dept. of Information Engineering, via Gradenigo Building) 43

Course requirements: Advanced calculus, and probability theory and random processes.

Examination and grading: A project assignment for students in groups of 2 requiring about 20 hours of work.

Aim: Complex networks are everywhere in real life. The most well known complex networks that we use everyday is the Internet and different forms of wireless networks. These are classical examples of Information Networks. Then there are various other types of networks such as biological networks in the human and animal bodies, social networks, citation networks, and many more. Modelling and analyzing such large-scale complex networks is a daunting task. Fortunately, mathematical tools such as Random Graphs and Stochastic Geometry allow us to construct simple but useful models of such networks, leading to tractable analysis and results that are surprisingly accurate for real world networks. Although these topics were historically developed from mathematical perspectives and used mostly in statistical physics, recent proliferation of large scale information and social networks has triggered a renewed research interest in using such tools for analyzing various forms of networks and developing design principles and resource allocation methods in complex new generation mobile communication networks for example. The applications of these tools are not just limited to a particular field, but have wide ranging applications in many areas including information network design and analysis, studying epidemic propagation, understanding social and biological networks etc. This course will deliver an introduction to the basic concepts and tools of random graphs and stochastic geometry. In addition, specific applications in wireless communication networks and the Internet, and multi-agent control networks in cyber-physical systems will be discussed.

Course contents:

- *Lecture 1: Introduction to Random Graphs for Networks:* Introduction to different types of Networks, the role of random graphs in studying networks, some Probability Theory preliminaries, basic models of random graphs such as the *Erdös-Rényi random graph* or *Poisson random graph* and the $G(n, p)$ random graph, properties of random graphs such as mean number of edges and mean degree, degree distribution, clustering coefficient
- *Lecture 2: Random graphs and their properties:* Components of a random graph such as the Giant Component and Small Components, phase transitions in a random graph and threshold functions: appearance of a subgraph, appearance of the giant component and appearance of a connected graph
- *Lecture 3: Random graphs and their properties (continued):* Sizes of the small components and their average behaviour,the complete distribution of the component sizes, path length behaviour in random graphs, the "small world effect", shortcomings of Random graph models in applications to real world networks

- *Lecture 4: Generalized Random Graphs:* Random graphs with general degree distributions, power-law distributions, size distributions of small components, giant component, other random graph models such as the random regular graphs
- *Lecture 5: Small world graphs and other random graph models:* The Small World model: a model that has high clustering coefficient and short average path length, degree distribution of the small world graph, clustering coefficient and the average path length of the small world graph, a short introduction to Exponential Random Graphs
- *Lecture 6: Basic Percolation Theory:* Introduction to lattice bond percolation theory: motivation and examples, formation of infinite-sized components in a percolated lattice and phase transition behaviour, phase transition in the random grid model: discrete percolation, percolation in interference limited networks
- *Lecture 7: Consensus and Gossip algorithms:* Introduction to consensus: linear consensus for distributed averaging, basic introduction to Graph theory and Laplacians, Perron-Frobenius theory for nonnegative matrices, consensus over random switching graphs, convergence results of linear consensus algorithms, gossip, randomized gossip and broadcast gossip algorithms and convergence
- *Lecture 8: Consensus and Gossip Algorithms:* Consensus over Markovian switching graphs, consensus for distributed estimation (filtering - linear and non-linear), consensus over wireless networks with various networking constraints and their effects on convergence
- *Lecture 9: Stochastic Geometry and its applications to wireless networks:* Introduction to Stochastic Geometry, Basic Point Process theory and properties, Point process transformations, Distributional Characterizations, General Point Processes, Cox Processes, Hard-core and Gibbs Processes
- *Lecture 10: Stochastic Geometry and its applications to wireless networks:* Sums and products of Point Processes, Moment Generating functional of sums over Poisson processes, Laplace functional and the probability generating functional for Point processes, Interference and outage probability characterization in Poisson Networks

References:

[1] Mark Newman, *Networks: An Introduction*, First Edition, Oxford University Press, UK, 2010.

[2] Bella Bollobas, *Random Graphs*, Second Edition, Cambridge Studies in Advanced Mathematics, Cambridge University Press, UK, 2001.

[3] M. Haenggi, *Stochastic Geometry for Wireless Networks*, Cambridge University Press, New York, 2013.

[4] M. Grossglauser and P. Thiran, “Networks out of Control: Models and Methods for Random networks”, Lecture notes, EPFL, 2012.

[5] M. Franceschetti and R. Meester, *Random Networks for Communication: ¿From Statistical Physics to Information Systems*, Cambridge University Press, UK, 2007.