Doctoral Course in Mathematical Sciences
Department of Mathematics
University of Padova

Doctoral Course in Mathematical Sciences

Catalogue of the courses 2016

Updated June 14, 2016

INTRODUCTION

The courses offered, for the year 2016, to the Graduate Students in Mathematical Sciences include courses taught by internationally recognized external researchers, who have accepted our invitation; such courses will not necessarily be offered again in the future years. Considering the wide impact of the content of theses courses, we emphasize the important for all graduate students to follow them.
The Faculty of the Graduate School could cancel courses with an excessively low number of registered students.

Also next year, beside the courses that our Doctoral Course directly offers, we have selected some courses of the Graduate School in Information Engineering of the University of Padova that we consider relevant also for our Course.

REQUIREMENTS FOR GRADUATE STUDENTS

With the advice of some Faculty member, all students are required to select some courses, either because they are linked with the curriculum of their present or planned research, or just to improve their knowledge of specific subject.

This year, considering the fact that courses may vary in duration, we have decided to indicate a mandatory minimum numbers of hour.
Therefore, students are required, within the **first two years (a half of the requirements within the first year)**, to follow and **pass the exam** of

Å**At least 2 courses of the PhD Programme**
Å other courses, <u>in addition to the two above</u>, in two curricula (**Computational Mathematics or Mathematics**) **or of the Doctoral Course**, with total commitment of **at least 64 hours**.

Students are encouraged to register for other courses; although to sit for the exam is not required for these courses, it is strongly advised. In all cases, students must participate with regularity to the activities of the courses they are registered to. At the end of the course the teacher will inform the Coordinators of the Curricula on the activities of the course and of the registered students.

<u>Institutional courses for Master of Science in ñMathematicsò</u>

Students have the possibility to attend, with acquisition of credits, the courses of the Master of Science in Mathematics.
The interest for these courses must be indicated by the Supervisor or a tutor. The Council will assign the number of hours that will be computed within the mandatory 64 hours.

<u>Courses attended in other Universities</u>

Students are allowed to take Ph.D. courses offered by PhD Programmes of other Universities. Acquisition of credits will be subject to approval of the Council.

HOW TO REGISTER TO COURSES

The online registration to courses has changed from last years, and allows students both to register and to cancel.
The registration is required for the attendance to all courses, independently of the intention to sit for the exam. The list of the courses can be found in the website of the Doctoral Course http://dottorato.math.unipd.it/ at the link Courses Registration (or directly at the address http://dottorato.math.unipd.it/registration/), filling the **online registration form** with all required data, and validating with the command ñRegisterò.
To acknowledge the registration, an email message will be sent to the address indicated in the registration form; this email message must be saved, since it is necessary for possible cancellation.

Registration for a course implies the agreement of the applicant to the participation.
Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except those that begin in January and February) using the link indicated in the email message of acknowledgment.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in the catalog, although part of activities in the Graduate School in Mathematics and thus offered to its students, are also open to all students, graduate students and researchers of all Graduate Schools and other universities.
For reasons of organization, external participants are required to **indicate their wish to participate at least two months before the beginning of the course for courses taking place from January 2016 and as soon as possible for courses that take place until December 2015, following the procedure described in the preceding paragraph.**
Possible **cancellation** to courses must also be notified.

Courses of the School

Courses of the “Computational Mathematics” area

Courses of the “Mathematics” area

3. Prof. Giovanna Carnovale
   Representation Theory of Groups **M-3**

4. Prof. Andrea D'Agnolo
   Topology 2 **M-4**

5. Prof. Christos Efthymiopoulos
   Applications of Canonical Perturbation Theory in Dynamical Astronomy **M-5**

6. Prof. Vladimir Mityushev
   Traditional and invisible composites: R-linear problem and its applications **M-6**

7. Prof. Simone Virili
   Selected Topics In Homological Algebra **M-7**

Courses in collaboration with the Doctoral School on "Information Engineering"

1. Proff. Tobias Damm, Michael Karow
   Applied Linear Algebra **DEI-1**

2. Prof. Subhrakanti Dey
   Random Graphs and Stochastic Geometry in Networks **DEI-2**

3. Prof. Paolo Favaro
   Inverse Problems in Imaging **DEI-4**

4. Prof. Lorenzo Finesso
   Statistical methods **DEI-5**

5. Prof. Fabio Marcuzzi
   Computational Inverse Problems **DEI-6**

6. Prof. Morten Gram Pedersen
   Mathematical modeling of cell Biology **DEI-7**

7. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-8**

8. Prof. Francesco Ticozzi
   Quantum Statistical Dynamics and Control **DEI-9**

Courses of the School

New trends in Numerical Analysis and Scientific Computing

Prof. Claude Brezinski[1]

[1] *Laboratoire Paul Painlevé, UMR CNRS 8524*
Université des Sciences et Technologies de Lille, France
Email: Claude.Brezinski@univ-lille1.fr

Timetable: 24 hours. Classes on Wednesday and Thursday, 11:30. First lecture on October 14, 2015, Torre Archimede, Room 2AB/45.

Course requirements: No special requirement is needed for this course. Only some fundamental knowledge of numerical analysis, but it could be acquired simultaneously with the lectures.

Examination and grading: Grading is based on homeworks or a written examination or both.

SSD: MAT/08 Numerical Analysis

Aim: The aim of the lectures is to introduce PhD students to some recent research subjects in numerical analysis (especially those related to approximation and numerical linear algebra) and to provide them the theoretical basis for their understanding. Applications will also be discussed. These lectures are intended to students and researchers in pure and applied mathematics, in numerical analysis, and in scientific computing.

Course contents: The various topics developed at different levels, will be

1. Formal orthogonal polynomials
   (a) Definition
   (b) Algebraic properties
   (c) Recurrence relation
   (d) Adjacent Families
2. Padé approximation
   (a) Definition and algebraic properties
   (b) Padé-type approximants
   (c) Connection to formal orthogonal polynomials
   (d) Recursive computation
   (e) Connection to continued fractions
   (f) Some elements of convergence theory
   (g) Applications
3. Krylov subspace methods
   (a) Definition
   (b) Lanczos method
   (c) Recurrence relations
   (d) Implementation
4. Extrapolation methods
   (a) Sequence transformations and convergence acceleration
   (b) What is an extrapolation method?
   (c) Various extrapolation methods
   (d) Vector sequence transformations
   (e) Applications

i. Treatment of the Gibbs phenomenon
ii. Web search
iii. Estimation of the error for linear systems
iv. Regularization of linear systems
v. Estimation of the trace of matrix powers
vi. Acceleration of Kaczmarz method
vii. Fixed point iterations
viii. Computation of matrix functions

References

[1] Lecture notes provided to the students following the courses.

[2] C. Brezinski, *Padé–Type Approximation and General Orthogonal Polynomials*, ISNM, vol. 50, Birkhäuser–Verlag, Basel, 1980.

[3] C. Brezinski, M. Redivo-Zaglia, *Extrapolation Methods. Theory and Practice*, North–Holland, Amsterdam, 1991. C. Brezinski, *Biorthogonality and its Applications to Numerical Analysis*, Marcel Dekker, New York, 1992.

[4] C. Brezinski, *Projection Methods for Systems of Equations*, North-Holland, Amsterdam, 1997.

[5] C. Brezinski, *Computational Aspects of Linear Control*, Kluwer, Dordrecht, 2002.

Hyperbolic PDES: a (locally smooth) introduction

Laura Caravenna[1], Carlotta Donadello[2]

[1] Universitá di Padova
Dipartimento di Matematica
Email: lcaraven@math.unipd.it

[2] Université de Franche-Comté
Laboratoire de Mathématiques, France
Email: Carlotta.Donadello@univ-fcomte.fr

Timetable: 22 hrs. First lecture on January 26, 2016, 10:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30

Course requirements: Before the course starts, we will meet interested students in order to adapt the program on audience's interests. We can for example discuss also numerical methods designed for conservation laws. In case of special interest, we can also include an overview on recent research lines.

Examination and grading: Either seminar or homework assignment.

SSD: MAT/05

Aim: This course focuses on hyperbolic PDES, with particular attention on conservation laws and transport. We aim at addressing basic topics accessible with essential background in calculus and functional analysis. The course should enable student to understand and approach relevant issues in the field.

Course contents: A tentative program includes the following. Scalar conservation laws: local existence, method of characteristics, loss of regularity and functional setting, Kruzhkov entropy conditions and equivalent formulations, global existence by vanishing viscosity, compensated compactness. For the scalar case, Riemann problem and other methods of constructing global entropy solutions: Glimm and wave front-tracking schemes. Examples. Classical theory on transport equation with smooth coefficients, Di Perna Lions results in Sobolev spaces, hints on Ambrosio's theory for vector field with bounded variation, an application of quasi-incompressible vector-fields.

Lie Algebras

Prof. Giovanna Carnovale[1]

[1] *Università di Padova, Dipartimento di Matematica*
Email: carnoval@math.unipd.it

Timetable: 25 hours. Lectures on Wednesday 11:30-13:15 and Thursday 11:30-13:15. First lecture on April 13, 2016 (to be confirmed, please contact the teacher a few days before the start of the course), Torre Archimede, Room 2AB/40.

Course requirements: Basic notions of linear algebra

Examination and grading: exercises

SSD: MAT/02

Aim: This course provides an introduction to Lie algebras and aims at presenting the classification of complex simple Lie algebras.

Course contents:

1. Basic notions. The adjoint representation and its subrepresentations. Derived subalgebra. Solvable and nilpotent Lie algebras. Nilpotent elements are ad-nilpotent.
2. Engel's theorem and Lie's theorem.
3. Irreducible representations of solvable Lie algebras. Schur's lemma.
4. Irreducible representations of sl(2,C). Uniqueness of the Jordan decomposition in End(V)
5. Killing form. Cartan's solvability criterion.
6. Cartan's semisimplicity criterion. Trace forms and Casimir element. Weyl's theorem.
7. Cartan subalgebras. Abstract Jordan decomposition.
8. The root space decomposition. sl2-triples.
9. Reductive Lie algebras. Root strings. Euclidean structure on the real span of roots.
10. Root systems and Weyl group.
11. Strategy for the classification of classical Lie algebras. Simple Lie algebras have irreducible root systems and viceversa.
12. Classical Lie algebras are simple (up to two cases).
13. Serre's theorem. Uniqueness of the semisimple Lie algebra associated with a root system. Uniqueness of the root system associated with a Lie algebra.

The fundamental Group in its different realizations

Prof. Bruno Chiarellotto[1]

[1] *University of Padova, Department of Mathematics*
Email: chiarbru@math.unipd.it

Timetable: 24 hrs. First lecture on October 13, 2015, 11:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic notion of topology, geometry and algebra.

Examination and grading: The exam will be taylored on the basis of the students which will attend this introductory course.

SSD: MAT/02 - MAT/03

Aim: to show how the notion of fundamental group can have different realization i.e. it can be built in different settings, not only in topology.

Course contents:
Starting from the topological definition using paths (and connecting it with singular homology theory), we will also introduce its definitions using coverings transformations. We then study the representations of the fundamental group. Using the riemann-hilbert correspondance we will see how such a representations are linked to connections. After that we will try to introduce the analogous definitions in arithmentic/algebraic terms. What should be the analogous of "covering" in the algebraic setting? What should be the analogous of representations of the fundamental group? This will lead to the definitions of "etale" coverings (In particular the fundamental group of a field as its galois group), to the theory of tannakian categories (those ones which are equivalent to some categories of representations). We then try to use the fundamental group as an invariant of the space: this will lead us to the theory of motives.

Mean field models, propagation of chaos and applications

Prof. Paolo Dai Pra[1], Prof. Markus Fischer[2]

[1] *University of Padova, Department of Mathematics*
Email: daipra@math.unipd.it

[2] *University of Padova, Department of Mathematics*
Email: fischer@math.unipd.it

Timetable: 24 hours, first lecture on April 21, 2016, 15:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: The course assumes the knowledge of measure theory and basic probability. The necessary notions of stochastic analysis will be briefly introduced in the first part of the course.

Examination and grading: Oral exam.

SSD: MAT/06 Probability and Mathematical Statistics

Aim:
The purpose is to introduce a class of dynamic models of many interacting components and derive their macroscopic behavior, i.e. the limit as the number of components tends to infinity. These models, characterized by the invariance with respect to permutations of the components and originally introduced as ”toy models” in Statistical Physics, have been intensively studied in the last decades for their applications to social and biological sciences. In particular, these models have been used in the study of large-scale socio-economic phenomena and of multicellular systems, e.g. networks of neurons. Moreover, the recent theories of mean-field control and games are built on these dynamics.

Course contents:

1. *INTRODUCTORY NOTIONS.* Basic results on stochastic differential equations, including models with discontinuous trajectories. Weak convergence of stochastic processes.
2. *PROPAGATION OF CHAOS.* Chaotic probabilities and propagation of chaos. The coupling approach. The martingale approach.
3. *APPLICATIONS.* Models of interacting agents. Neuronal networks. Control of mean-field systems.

Courses of the “Computational Mathematics” area

Percolation

Prof. Raphael Cerf

Université de Paris Sud
Email: raphael.cerf@gmail.com

Timetable: 12 hrs, first lecture on April 5, 2016, 13:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: Standard knowledge of Probability and measure theory.

Examination and grading: Seminar on a subject assigned by the Instructor.

SSD: MAT/06

Aim: Percolation is one of the macroscopic phenomena that can have bee extensively studied in rigorous Statistical Physics. This course presents the basic mathematical theory of percolation.

Course contents:
This is an introductory course on the percolation model on the cubic lattice. The percolation model is the simplest model presenting a phenomenon of phase transition. The basic tools to study percolation and the fundamentals results will be presented.

Convex polytopes

Prof. Michele Conforti

Università degli Studi di Padova
Dipartimento di Matematica
Email: conforti@math.unipd.it

Timetable: 12 hrs. First lecture on January 18, 2016, 14:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic knowledge of the theory of linear programming and polyhedra: this can be achieved by reading Chapters 7 and 8 in [1] or Chapter 3 in [2] before the beginning of the course.

Examination and grading: Written exam.

SSD: MAT/09.

Aim: Polytopes arise in optimization, but have been studied for long time, e.g., in Physics, Chemistry and Biology. We survey some classical and very recent results on the structure of polytopes.

Course contents:

1. Definition and examples of polytopes: 3-dimensional polytopes (including Platonic solids), cyclic polytopes, permutahedron, combinatorial polytopes.
2. Representations of polyhedra: $\mathcal{H}$-polyhedra and $\mathcal{V}$-polyhedra.
3. Affine hull, recession cone, lineality space.
4. Facets, vertices and extreme rays.
5. Face lattice of a polytope.
6. Skeleton of a polytope and Balinski's theorem.
7. Diameter of a polytope and the Hirsch conjecture: sub-exponential upper-bounds, correctness of the Hirsch conjecture for 0/1 polytopes, counterexample to the Hirsch conjecture (sketch), polynomial upper bound for totally unimodular matrices.

References:

[1] A. Schrijver, *Theory of Linear and Integer Programming*, Wiley, 1986.

[2] M. Conforti, G. Cornuéjols, G. Zambelli, *Integer Programming*, Springer, 2014.

Epidemic stochastic models

Carles Rovira Escofet[1]

[1] *Departament de Probabilitat, Logica i Estadistica, Universitat de Barcelona, Spain*
Email: carles.rovira@ub.edu

Timetable: 10 hrs. First lecture on April 27, 2016, 11:00, (dates already fixed, see the calendar) Torre Archimede, Room 2BC/30.

Course requirements: Probability, Ordinary differential equations and Stochastic processes.

Examination and grading: according with the teacher

SSD: MAT/06

Aim: We present an introduction to the formulation of some types of deterministic and stochastic epidemic models. We begin with the well-known deterministic SIS and SIR epidemic models. we also consider some delay models in Mathematical Biology. Two different types of stochastic models are presented: discrete time Markov chain and stochastic differential equations. We ends with a discussion about stability.

Course contents:

1. Basic deterministic models in epidemiology: SIR, SIS, SIRS
2. Delay deterministic models
3. Discrete time stochastic models in epidemiology
4. Continuous time stochastic models in epidemiology with or without delays
5. Global stability

Local martingales and the martingale property

Prof. Johannes Ruf

University College London
Email: j.ruf@ucl.ac.uk

Timetable: 12 hrs, first lecture on April 6, 2016, 16:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: A background in stochastic calculus

Examination and grading: Homework problems and an oral presentation (only pass-fail grades are awarded)

SSD: MAT/06

Aim: Making participants familiar with local martingales and methods to decide on the martingale property.

Course contents: Examples for strict local martingales, including local martingales with jumps. How to generate strict local martingales in a systematic way. Review of different methods and their proofs to decide on martingale property; in particular, methods based on weak tails and Novikov-type conditions.

References to start reading, *available on http://www.oxford-man.ox.ac.uk/ jruf/:*

1. Hulley Ruf: Weak tail conditions for local martingales (2015).
2. Blanchet Ruf: A weak convergence criterion for constructing changes of measure, Stochastic Models (2015).
3. Ruf: The martingale property in the context of stochastic differential equations, Electronic Communications in Probability, Volume 20, Issue 34 (2015).

Topics in Numerical Probability with applications to advanced Quantitative Finance

Prof. Erik Schlogl[1]

[1] *University of Technology Sydney*
UTS Business School
Email: Erik.Schlogl@uts.edu.au

Timetable: 10 hrs, first lecture on June 17, 2016, 10:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements:
Standard probability theory and stochastic calculus

Examination and grading:
Project

SSD: INF/01 Computer Science, MAT06 Probability

Title:
Topics in Numerical Probability with applications to advanced Quantitative Finance

Course contents:
Session 1

Gram/Charlier Type A Series Expansions and an application to market-implied distributions and option pricing

If a probability distribution is sufficiently close to a normal distribution, its density can be approximated by a Gram/Charlier Series A expansion. In finance, this has been used to fit risk-neutral asset price distributions to prices observed on the market, ensuring an arbitrage-free interpolation of implied volatilities across exercise prices. We will discuss the option pricing formula in terms of the full (untruncated) series and consider a fitting algorithm, which ensures that a series truncated at a moment of arbitrary order represents a valid probability density. Generalising the Gram/Charlier Series A approach to the multiperiod, multivariate case, a model calibrated to standard option prices is developed, in which a large class of exotic payoffs can be priced in a single formula.

Outline:

1. Background and some useful lemmas 2. The Gram/Charlier expansion 3. Modelling asset price distributions 4. Extending the model to multiple assets and maturities 5. Further directions and applications

Session 2

Generic and object-oriented programming techniques for Monte Carlo simulation in C++

In many numerical probability applications, MC simulation is the easiest numerical method to implement, and for highdimensional problems it is often the most efficient. It is also a field of application where generic and object-oriented programming techniques assist in building a powerful toolbox. Following a philosophy that ideally every bit of functionality should be implemented once (only) in a well-designed library, I will discuss how building blocks can be created, representing the generic Monte Carlo algorithm, various control variate techniques, the Longstaff/Schwartz evaluation of optimal stopping problems, and quasi-random number generation. In addition to their applicability to more straightforward cases, these same building blocks can be combined, for example, to analyse complex financial instruments like Bermudan products which combine equity, interest rate and currency risk under any risk-neutral or real-world probability measure, or to combine control variate techniques with quasi-random number generation, in each case without the need to re-code the building blocks.

Outline:

1. Background & approach 2. Generic algorithm implementation 3. Extensions for variance reduction 4. Early exercise 5. Varying the building blocks Embarrassingly parallel? Parallel computing issues

Sessions 3 & 4 Fundamentals of Credit Risk Modelling for Counterparty Credit Risk Assessment and Valuation

This talk will cover the main concepts of counterparty credit risk in derivative financial instruments, i.e. expected exposure (EE), potential future exposure (PFE), credit valuation adjustment (CVA), and related values. It will focus on those aspects of mathematical modelling of credit risk needed to calculate counterparty credit risk as represented by these concepts, and discuss how CVA can be "marked-to-market" based on risk-neutral default probabilities implied by market credit spreads. In a simple, analytically tractable example, these concepts and calculations will be illustrated in a spreadsheet implementation. Outline: 1. Defining and modelling counterparty risk 2. Measuring counterparty risk 3. Obtaining implied default probabilities 4. Counterparty risk calculations

Topics in Stochastic Analysis

Prof. Tiziano Vargiolu[1]

[1] Università di Padova
Dipartimento di Matematica Pura ed Applicata
Email: vargiolu@math.unipd.it

Calendario: 10 hrs. First lecture on October 20, 2015, 10:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Prerequisiti: A previous knowledge of the basics of continuous time stochastic analysis with standard Brownian motion, i.e. stochastic integrals, Itô formula and stochastic differential equations, as given for example in the master course "Analisi Stocastica".

Tipologia di esame: Seminar

SSD: MAT/06

Programma: The program will be fixed with the audience according to its interests. Some examples could be:

- continuous time stochastic control;
- Levy processes;
- numerical methods;
- stochastic control.

Numerical methods for Ordinary Differential Equations

Prof. Marino Zennaro[1]

[1] *University of Trieste, Department of Mathematics and Geosciences*
Email: zennaro@units.it

Timetable: 12 hrs. First lecture on January 12, 2016, 15:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: it is advisable to have attended a basic course in Numerical Analysis.

Examination and grading: Written exam.

SSD: MAT/08 Numerical Analysis

Aim: We present basic numerical methods for initial value problems in ordinary differential equations and we analyse their convergence properties.

Course contents:

- Existence and uniqueness of the solution and continuous dependence on the data for the initial value problem $y'(x) = f(x, y(x)), y(x_0) = y_0$.
- Classical Lipschitz constant and right hand side Lipschitz constant.
- General one-step methods; explicit and implicit Runge-Kutta methods.
- Definition of local truncation and discretization error for one-step methods and definition of consistency of order p.
- Convergence theorem with order p for one-step methods. Order conditions for Runge-Kutta methods. Order barriers for explicit and implicit methods.
- Variable stepsize implementation. Embedded pairs of methods of Runge-Kutta-Fehlberg type.

References:

1. E. Hairer, S.P. Norsett, G. Wanner: Solving Ordinary Differential Equations I, Nonstiff Problems, Springer-Verlag, Berlin, 1993.
2. J.C. Butcher: Numerical methods for ordinary differential equations. Second edition, John Wiley & Sons, Ltd., Chichester, 2008.
3. Lecture notes by the professors.

Courses of the “Mathematics” area

Arithmetic D-modules

Prof. Tomoyuki Abe[1]

[1]Kavli Institute for the Physics and Mathematics of the Universe
Email: tomoyuki.abe@ipmu.jp

Timetable: 12 hrs. First lecture on October 21, 2015, 14:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/02-03

Aim:

Course contents:

Thanks to hard works by many people including Prof. Baldassarri and Chiarellotto, most of the fundamental properties of p-adic cohomology theory are well-established. However, as in the etale cohomology theory, six functor formalism for such theory had been required for some applications such as Langlands correspondence. Following algebraic D-module theory, Berthelot introduced so called arithmetic D-modules, and showed us a road map towards constructing such theory. In this lecture, I'd like to explain the current status of the theory including D. Caro's foundational contribution to the theory as well as my recent results.

Spectral stability of differential operators

Prof. Victor I. Burenkov[1]

[1] Peoples' Friendship University of Russia
Department of of Mathematical Analysis and the Theory of Functions Moscow, Russia
Email:

Timetable: 12 hrs. First lecture on January 11, 2016, 11:00, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30

Course requirements:

Examination and grading:

SSD: MAT/05

Aim:

Course contents:

1. Unbounded closed operators
2. Weak derivatives and the weak Laplacian
3. Sobolev spaces and the Laplacian graph spaces
4. Further properties of the weak Laplacian and the Laplacian graph spaces
5. Classes of open sets
6. Traces of functions in Sobolev spaces
7. Closure of the Dirichlet Laplacian
8. Dirichlet Laplacian on an arbitrary open set
9. Neumann Laplacian on an arbitrary open set
10. Unbounded symmetric and self-adjoint operators
11. Quadratic forms. Friedrichs extensions of non-negative symmetric operators
12. Fractional powers of non-negative self-adjoint operators
13. Spectrum of non-negative self-adjoint operators with compact resolvents
14. Minimax principle for the eigenvalues of non-negative self-adjoint operators with compact resolvents
15. Transition operators. General spectral stability theorems for non-negative self-adjoint operators with compact resolvents
16. Spectral stability of the Dirichlet Laplacian
17. Spectral stability of the Neumann Laplacian
18. Spectral stability of the Robin Laplacian
19. Further spectral stability results

Representation Theory of Groups

Prof. Giovanna Carnovale[1]

[1] *Università di Padova, Dipartimento di Matematica*
Email: carnoval@math.unipd.it

Timetable: 14 hours. Lectures on Wednesday 11:30-13.15 and Thursday 11:30-13.15. First lecture on March 2, 2016, Torre Archimede, Room 2AB/40.

Course requirements: Basic notions of linear algebra and of group theory

Examination and grading: exercises

SSD: MAT/02

Aim: This course provides an introduction to the representation theory of groups, with focus on character theory for complex representations of finite groups.

Course contents:

1. Basic notions of representation theory: representations, irreducible representations, completely reducible representations, indecomposable representations.
2. Tensor products, exterior and symmetric powers, duals, representiation structure on Hom spaces. Schur's lemma.
3. Characters and their main properties. Orthogonality relations. Isotypical components. Decomposition of the regular representation.
4. Complex irreducible characters are an orthonormal basis for the space of central functions.
5. Construction of irreducible representations for abelian groups. How to enumerate complex 1-dimensional representations in a finite group. Induced representations and their character.
6. Frobenius reciprocity. Algebraic integers. Dimension of an irreducible representation.
7. Frobenius-Schur indicator. Enumerating involutions in a finite group. Compact groups and their representation theory.

Topology 2

Andrea D'Agnolo[1]

[1] Universitá di Padova
Dipartimento di Matematica
Email: dagnolo@math.unipd.it

Timetable:
for information regarding the timetable of the classes please contact prof. Andrea D'Agnolo (dagnolo@math.unipd.it) before October 15th.

Course requirements:

Examination and grading:

SSD: MAT/03-MAT/05

Aim: see http://tiny.cc/topologia

Course contents: Algebraic Topology is usually approached via the study of the fundamental group and of homology, defined using chain complexes, whereas, here, the accent is put on the language of categories and sheaves, with particular attention to locally constant sheaves.

Sheaves on topological spaces were invented by Jean Leray as a tool to deduce global properties from local ones. This tool turned out to be extremely powerful, and applies to many areas of Mathematics, from Algebraic Geometry to Quantum Field Theory.

On a topological space, the functor associating to a sheaf the space of its global sections is left exact, but not right exact in general. The derived functors are cohomology groups that encode the obstructions to pass from local to global. The cohomology groups of the constant sheaf are topological (and even homotopical) invariants of the space, and we shall explain how to calculate them in various situations.

Applications of Canonical Perturbation Theory in Dynamical Astronomy

Prof. Christos Efthymiopoulos[1]

[1] Research Director at the
Research Center for Astronomy and Applied Mathematics, Academy of Athens
Email: cefthim@Academyofathens.gr

Timetable: 14 hrs. First lecture on November 6, 2015, 10:30, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Examination and grading: Students will be assigned with small individual projects, aiming to develop their ability to make computations in canonical perturbation theory. A final written report on each project will have to be delivered upon the course's end. The evaluation will be based on the written reports.

SSD: MAT/07 - Mathematical Physics

Aim: The course aims to provide an application-oriented introduction to the techniques and tools of canonical perturbation theory. To this end, the course presents examples of concrete series computations allowing to draw information about the dynamics in some systems encountered in dynamical astronomy. In particular, the examples dealt with are:

1. the spin-orbit problem and the location of secondary resonances in the case of planets or satellites trapped in a spin-orbit resonance;
2. the secular dynamics in a fictitious planetary system composed of one star and two planets;
3. center (normally hyperbolic) manifold computations near the Lagrangian point L1 in the circular restricted three body problem (applying to the orbits of spacecrafts exploiting the so-called "space manifold dynamics");
4. adiabatic invariants and the motion of charged particles in magnetospheres.

All examples will be substantiated by computer-algebraic demonstrations during the lectures.

Course contents:

1. Introduction to the canonical formalism. Canonical transformations with Lie series. Hamiltonian normal forms. Small divisors. Non-resonant series. Resonances. Asymptotic behavior.
2. Simple examples of application: the time-modulated perturbed pendulum problem, Hamiltonian systems of two or three degrees of freedom with a convex integrable part. Estimates on chaos thresholds and diffusion.
3. Spin-orbit problem: formulation, 1:1 (Earth-Moon) and 3:2 (Sun-Mercury) resonances. Birkhoff normal form around the 1:1 resonance. Location of the 3:1 secondary resonance.
4. Secular dynamics: the Hamiltonian of the three-body problem in the Poincaré variables. Modified Delaunay variables. Disturbing function, expansion of. Averaging over short period terms. Determination of the secular solution.
5. Center manifold computations in the CRTBP: computation of the hyperbolic normal form. Reduction to the center manifold. Location and stability of the halo family of orbits.
6. Adiabatic invariants: magnetic bottle Hamiltonians. Normalization in the case of a nilpotent kernel. Computation of mirror frequencies. Resonances and the onset of global chaos.

Traditional and invisible composites: R-linear problem and its applications

Prof. Vladimir Mityushev[1]

[1] Dept of Comp. Sciences and Comp. Methods, Pedagogical University, ul. Podchorazych 2, Krakow 30-084, POLAND
Email: http://mityu.up.krakow.pl

Timetable: 8 hours. First lecture on July 22, 2016, 10:00 (following lectures will be agreed with the teacher), Torre Archimede, Room 2BC/30.

Course requirements: No special requirement is needed for this course. Only some fundamental knowledge of calculus and introductory knowledge of complex analysis.

Examination and grading: Grading is based on an oral examination.

SSD: MAT/05

Aim: The aim of the lectures is to introduce PhD students to some recent researches in applied mathematics devoted to composites; to provide them the method of complex potentials and constructive homogenization based on asymptotic analysis; to introduce PhD students to recent mathematical study of invisible materials. These lectures are intended to students and researchers in pure and applied mathematics and in scientific computing (symbolic computations).

Course contents:
Statement of the $\mathbb{R}$-linear problem and its relations to the classic boundary value problems. Riemann-Hilbert problem.
Application to composites. Constructive homogenization.
Representative volume element.
Asymptotic methods to calculate the effective constants.
Schwarz's alternating method and functional equations.
Implementation of the method and symbolic-numerical computations.
Mathematical foundations of invisible materials.

References

[1] O'Neill, J., Selsil, Ö., McPhedran, R.C., Movchan, A.B., Movchan, N.V. (2015) Active cloaking of inclusions for flexural waves in thin elastic plates *Q J Mechanics Appl Math*, doi:10.1093/qjmam/hbv007.

[2] Mityushev V., Rogosin S. *Constructive Methods for Linear and Nonlinear Boundary Value Problems for Analytic Functions. Theory and Applications*, Chapman & Hall / CRC, Monographs and Surveys in Pure and Applied Mathematics, Boca Raton, 2000, 283 pp.

[3] Mityushev V., Nawalaniec W. Basic sums and their random dynamic changes in description of microstructure of 2D composites, Computational Materials Science, Computational Materials Science, 97, (2015) 64-74.

[4] $en.wikipedia.org/wiki/$ Metamaterial cloaking

[5] $wolfram.com/index.php$ Mathematica

Selected Topics In Homological Algebra

Simone Virili[1]

[1] *Departamento de Matemáticas, Autonomous University of Barcelona, Spain*
Email:

Timetable: 10 hrs. First lecture on May 2, 2016, 11:30, (dates already fixed, see the calendar) Torre Archimede, Room 2BC/30.

Course requirements: Some knowledge of category theory and of Abelian categories. Basic facts from ring and module theory will be of great help.

Examination and grading: Lists of exercises will be assigned weekly. There will be also a final oral examination.

SSD: MAT/03

Aim: The aim of this lecture series is to give an overview of the classical theory of localizations of Grothendieck categories (due to Grothendieck, Gabriel, Popescu, and others), and to underline similarities and differences with the theory of localization of derived categories (giving an overview of results of Neeman, Krause, Balmer, Stevenson, Rickard and others). To do so, we will need to develop along the way some of the basic techniques of triangulated and derived categories.

Course contents:

1. Grothendieck Categories:
   (a) Grothendieck categories, definitions and basic facts;
   (b) torsion theories and localization of Grothendieck categories;
   (c) the Gabriel filtration and the Gabriel dimension;
   (d) Gabriel's classification of hereditary torsion theories.
2. Derived and triangulated categories:
   (a) the axioms of a triangulated category;
   (b) our main example: the derived category of a Grothendieck category;
   (c) derived functors;
   (d) homotopy (co)limits and resolution of unbounded complexes.
3. Support theory for triangulated categories:
   (a) tensor-triangulated categories;
   (b) localization of triangulated categories;
   (c) the Balmer spectrum of triangulated ideals;
   (d) the classification of localizing subcategories in a triangulated category.

References:

1. P. Balmer, The spectrum of prime ideals in tensor triangulated categories
2. M. Bkstedt, A. Neeman, Homotopy limits in triangulated categories

Courses in collaboration with the Doctoral School on "Information Engineering"

Applied Linear Algebra

Prof. Tobias Damm[1], Prof. Michael Karow[2]

[1] *Technische Universität Kaiserslautern, Germany*
Email: damm@mathematik.uni-kl.de

[2] *Technische Universität Berlin, Germany*
Email: karow@math.tu-berlin.de

Timetable: 16 hours. First lecture on March, Monday 21st 4:30 am. Second Lecture on March, Wednedsday 23rd, 2.30 pm. All other 6 lectures on Wednesday (2.30 pm) and Friday (10.30 am) starting from March, Wednesday 30th 2016 (Dept. of Information Engineering, Via Gradenigo 6/a, Padova)

Course requirements: A good working knowledge of basic notions of linear algebra as for example in [1]. Some proficiency in MATLAB.

Examination and grading: Grading is based on homeworks or a written examination or both.

Aim: We study concepts and techniques of linear algebra that are important for applications with special emphasis on the topics *low rank approximation* and *matrix equations and inequalities*. A wide range of exercises and problems will be an essential part of the course and constitute homework required to the student.

Course contents:

- Review of some basic concepts of linear algebra and matrix theory
- The singular value decomposition and applications
- Krylov subspaces
- Matrix equations and matrix inequalities
- Sylvester and Lyapunov equations, Riccati equation, linear matrix inequalities (LMIs)

References:

[1] Gilbert Strang's linear algebra lectures, from M.I.T. on You Tube

[2] Notes from the instructors

Random Graphs and Stochastic Geometry in Networks

Prof. Subhrakanti Dey[1]

[1] *Signals and Systems, Uppsala University, Sweden*
Email: Subhra.Dey@signal.uu.se

Timetable: 20 hrs. Class meets every Monday and Wednesday from 10:30 to 12:30. First lecture on Wednesday, March 30th, 2016. Room DEI/G (3-rd floor, Dept. of Information Engineering, via Gradenigo Building) 43

Course requirements: Advanced calculus, and probability theory and random processes.

Examination and grading: A project assignment for students in groups of 2 requiring about 20 hours of work.

Aim: Complex networks are everywhere in real life. The most well known complex networks that we use everyday is the Internet and different forms of wireless networks. These are classical examples of Information Networks. Then there are various other types of networks such as biological networks in the human and animal bodies, social networks, citation networks, and many more. Modelling and analyzing such large-scale complex networks is a daunting task. Fortunately, mathematical tools such as Random Graphs and Stochastic Geometry allow us to construct simple but useful models of such networks, leading to tractable analysis and results that are surprisingly accurate for real world networks. Although these topics were historically developed from mathematical perspectives and used mostly in statistical physics, recent proliferation of large scale information and social networks has triggered a renewed research interest in using such tools for analyzing various forms of networks and developing design principles and resource allocation methods in complex new generation mobile communication networks for example. The applications of these tools are not just limited to a particular field, but have wide ranging applications in many areas including information network design and analysis, studying epidemic propagation, understanding social and biological networks etc. This course will deliver an introduction to the basic concepts and tools of random graphs and stochastic geometry. In addition, specific applications in wireless communication networks and the Internet, and multi-agent control networks in cyber-physical systems will be discussed.

Course contents:

- *Lecture 1: Introduction to Random Graphs for Networks:* Introduction to different types of Networks, the role of random graphs in studying networks, some Probability Theory preliminaries, basic models of random graphs such as the *Erdös-Rényi random graph* or *Poisson random graph* and the $G(n, p)$ random graph, properties of random graphs such as mean number of edges and mean degree, degree distribution, clustering coefficient
- *Lecture 2: Random graphs and their properties:* Components of a random graph such as the Giant Component and Small Components, phase transitions in a random graph and threshold functions: appearance of a subgraph, appearance of the giant component and appearance of a connected graph
- *Lecture 3: Random graphs and their properties (continued):* Sizes of the small components and their average behaviour,the complete distribution of the component sizes, path length behaviour in random graphs, the “small world effect”, shortcomings of Random graph models in applications to real world networks

Inverse Problems in Imaging

Prof. Paolo Favaro[1]

[1] *Institute of Computer Science and Applied Mathematics, University of Bern, Switzerland*
Email: favaro@iam.unibe.ch

Timetable: 16 hrs. Class meets every Tuesday and Thursday from 14:30 to 16:30. First lecture on Tuesday, January 12, 2016. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: Basic notions of probability theory, linear algebra, calculus and differential equations, knowledge of computing programming using Matlab.

Examination and grading: A short programming assignment (30% of the final grade) and a final exam (70% of the final grade).

Aim: The course provides an introduction to inverse problems in image processing, such as denoising, deblurring, and blind deconvolution. The main focus is to present analytical and numerical tools for solving inverse problems and to illustrate connections to several other bilinear problems in imaging, such as independent component analysis, dictionary learning/sparse coding, and matrix factorization.

Course contents:

- *The fundamentals of inverse problems:* image formation models, illposedness and ill-conditioning, denoising and deblurring problems, apriori information, regularization techniques, a primer on numerical solvers.
- *The Bayesian formulation and methodologies:* generative models, image priors, inference methods (e.g., Conditional Mean, Maximum a Posteriori)
- *Numerical methods:* a primer on convex optimization, descent algorithms, the Primal-Dual method, Expectation-Maximization, Variational Bayes, Majorization Minimization.
- *Advanced problems and techniques in imaging:* sparsity-based reconstructions, total variation denoising and deblurring, blind deconvolution, relations to other bilinear problems in imaging.

References:

[1] M. Bertero and P. Boccacci, *Introduction to Inverse Problems in Imaging*, Inst. of Physics Publications 1998.
[2] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press 2004.
[3] R. T. Rockafellar, *Convex analysis*, Princeton University Press 1996.
[4] C. M. Bishop, *Pattern Recognition and Machine Learning*, Springer 2012.
[5] A. Chambolle and T. Pock, *A first-order primal-dual algorithm for convex problems with applications to imaging*, J. of Mathematical Imaging and Vision, 40(1), 120-145, 2011.
[6] D. Wipf and H. Zhang, *Revisiting Bayesian Blind Deconvolution*, Energy Minimization Methods in Computer Vision and Pattern Recognition, 40-53, 2013.
[7] T. Chan and C. K. Wong, *Total variation blind deconvolution*, IEEE Transactions on Image Processing, 7(3), 370-375, 1998.
[8] D. Perrone and P. Favaro, *Total variation blind deconvolution: The devil is in the details*, Computer Vision and Pattern Recognition, 2909?2916, 2014.

Presentation material will be provided at the course and will be available on the course website together with additional research articles and books.

Statistical methods

Prof. Lorenzo Finesso[1]

[1] *Istituto di Elettronica e di Ingegneria dell'Informazione e delle Telecomunicazioni, IEIIT-CNR, Padova*
Email: lorenzo.finesso@unipd.it

Timetable: 24 hrs (two lectures of two hours each per week). Class meets every Monday and Wednesday from 14:30 to 16:30, starting on Monday, April 18-th, 2016. Meeting Room DEI/G (3-rd floor, Dept. of Information Engineering, via Gradenigo Building).

Course requirements: Basics of probability, basics of linear algebra.

Examination and grading: homework assignments and take-home exam.

Aim: The course will present a small selection of linear statistical techniques which are widespread in applications. The unifying power of the information theoretic point of view will be stressed.

Course contents:

Background material. The noiseless source coding theorem will be quickly reviewed in order to introduce the basic notions of entropy of a probability measure and I-divergence (a.k.a. relative entropy, Kullback-Leibler distance) between two probability measures.

Divergence minimization problems. Three divergence minimization problems will be posed and, via examples, they will be connected with basic methods of statistical inference: ML (maximum likelihood), ME (maximum entropy), and EM (expectation-maximization).

Multivariate analysis methods. The three standard multivariate methods, PCA (Principal component analysis), Factor Analysis, and CCA (Canonical Correlations analysis) will be reviewed and their connection with divergence minimization discussed. Applications of PCA to least squares (PCR principal component regression, PLS Partial least squares). Approximate matrix factorization and PCA, with a brief detour on the approximate Nonnegative Matrix Factorization (NMF) problem.

EM methods. The Expectation-Maximization method will be introduced as an algorithm for the computation of the Maximum Likelihood (ML) estimator with partial observations (incomplete data) and interpreted as an alternating divergence minimization algorithm (à la Csiszár Tusnády).

Applications to stochastic processes. Derivation of Burg spectral estimation method as solution of a Maximum Entropy problem. Introduction to HMM (Hidden Markov Models). Maximum likelihood estimation for HMM via the EM method.

References: A set of lecture notes and a list of references will be posted on the web site of the course.

Computational Inverse Problems

Prof. Fabio Marcuzzi[1]

[1] University of Padova
Department of Mathematics
Email: marcuzzi@math.unipd.it

Timetable: 16 hrs (2 two-hours lectures per week): Classes on Monday and Wednesday, 10:30 - 12:30. First lecture on Monday February 22th, 2016. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: basic notions of linear algebra and, possibly, numerical linear algebra. The examples and homework will be in Python (the transition from Matlab to Python is effortless).

Examination and grading: Homework assignments and final test.

SSD: MAT/08

Aim: We study numerical methods that are of fundamental importance in computational inverse problems. Real application examples will be given for distributed parameter systems. Computer implementation performance issues will be considered also.

Course contents:

- definition of inverse problems, basic examples and numerical difficulties.
- numerical methods for QR and SVD and their application to the squareroot implementation in PCA, least-squares, model reduction and Kalman filtering; recursive least-squares;
- regularization methods;
- numerical algorithms for nonlinear parameter estimation: Gauss-Newton, Levenberg-Marquardt;
- examples with distributed parameter systems;
- HPC implementations.

References:

[1] F.Marcuzzi "Analisi dei dati mediante modelli matematici",
`http://www.math.unipd.it/~ marcuzzi/MNAD.html`

Mathematical modeling of cell Biology

Prof. Morten Gram Pedersen[1]

[1] *Dept. of Information Engineering*
University of Padova
Email: pedersen@dei.unipd.it

Timetable: 20 hrs (2 two-hours lectures per week). Class meets every Monday and Wednesday from 14:30 to 16:30. First lecture on Monday, October 12, 2015. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: Basic courses of linear algebra and ODEs. Basic experience with computer programming. Knowledge of cellular biology is not required.

Examination and grading: Final project.

Aim: The aim of this course is to provide an introduction to commonly used mathematical models of cellular biology. At the end of the course, the students should be able to build models of biological processes within the cell, to simulate and analyze them, and to relate the results back to biology. The focus will be on electrical activity and calcium dynamics in neurons and hormone-secreting cells, but will also discuss models of other cellular processes occurring in other cell types.

Topics: Biochemical reactions; Ion channels, excitability and electrical activity; Calcium dynamics; Intercellular communication; Spatial and stochastic phenomena (if time allows); Contractions in muscles; Circadian rhythms; Qualitative analysis of nonlinear differential equations.

References:
The following books will provide the core material, which will be supplemented by research articles:

1. C.P. Fall, E.S. Marland, J.M. Wagner, J.J. Tyson. *Computational Cell Biology*. Springer, NY, USA (2002).
2. J. Keener, J. Sneyd. *Mathematical Physiology*. Springer, NY, USA (2004).

Applied Functional Analysis and Machine Learning

Prof. Gianluigi Pillonetto[1]

[1] *Dept. of Information Engineering, University of Padova*
Email: giapi@dei.unipd.it

Timetable: 28 hrs (2 two-hours lectures per week): Classes on Tuesday and Thursday, 10:30 - 12:30. First lecture on Tuesday October 13th, 2015. Sala Riunioni 318 DEI/G 3-rd oor, via Gradenigo 6).

Course requirements: The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. The arithmetic of complex numbers and the basic properties of the complex exponential function. Some elementary set theory. A bit of linear algebra. All the necessary material can be found in W. Rudin's book Principles of Mathematical Analysis (3rd ed., McGraw-Hill, 1976). A summary of the relevant facts will be given in the first lecture.

Examination and grading: Homework assignments and final test.

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

Course contents:

1. *Review of some notions on metric spaces and Lebesgue integration*: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.
2. *Banach and Hilbert spaces*: Normed spaces and Banach spaces. Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces. Hilbert adjoint operator. Self-adjoint operators, unitary operators.
3. *Compact linear operators on normed spaces and their spectrum*: Spectral properties of bounded linear operators. Compact linear operators on normed spaces. Spectral properties of compact linear operators. Spectral properties of bounded self-adjoint operators, positive operators, operators defined by a kernel. Mercer Kernels and Mercer's theorem.
4. *Reproducing kernel Hilbert spaces, inverse problems and regularization theory*: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Basic concepts of convex analysis. Primal and dual formulation of loss functions. Regularization networks. Support vector regression and classification. Support vector classification.

References:

[1] W. Rudin. Real and Complex Analysis, McGraw Hill, 2006.

[2] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons, 1978.

[3] G. Wahba. Spline models for observational data. SIAM, 1990.

[4] C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006.

[5] R.T. Rockafellar. Convex analysis. Princeton University Press, 1996.

Quantum Statistical Dynamics and Control

Prof. Francesco Ticozzi[1]

[1] Dept. of Information Engineering
University of Padova
Email: ticozzi@dei.unipd.it

Timetable: 16 hrs. Class meets every Monday and Wednesday from 10:30 to 12:30. First lecture on Wednesday, June 1st, 2016. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: Linear algebra and probability theory.

Examination and grading: Homeworks.

Aim: The course starts by providing an introduction to (elementary) quantum mechanics from the viewpoint of probability theory, accessible without any quantum mechanics background. The second part of the course is devoted to the definition and the study of quantum dynamical semigroups. This class of dynamics is widely used to model physical systems of interest in quantum information and control. In the last part of the course, some applications, illustrating the use of the mathematical tools developed, will be presented, including problems of information encoding and state preparation for finite-dimensional systems.

Topics:

- **Quantum Theory as a Probability Theory:** Densities, observable quantities, measurements in a non-commutative setting. Composite systems and entanglement. Partial trace and marginal densities. (4h)
- **Quantum Dynamical Systems:** Unitary dynamics, open quantum systems and quantum operations. Kraus representation theorem. Examples for two-level systems. Quantum dynamical semigroup and completely positive generators, and their representations. (4h)
- **Stability Analysis:** Basic stability properties, existence and structure of the invariant sets. Elements of Lyapunov-type analysis and natural Lyapunov functions. (4h)
- **Applications:** Noiseless encodings of quantum information; Preparation of states, subspaces and subsystems; Feedback master equations and their control. (4h)

References:

- Lecture notes and supplementary material provided by the instructor;
- A good introductory reference to modern quantum mechanics for finite dimensional systems is contained in the first chapters of: M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum information (Cambridge, 2000).
- A "classic" reference for continuous-time dynamical semigroups is: R. Alicki and K. Lendi, Quantum Dynamical Semigroups and Applications. Springer-Verlag, Berlin, 1987.

Calendar

The calendar is not completely filled in. Updated June 14, 2016

ottobre 2015

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
27	28	29	30	1	2	3
4	5	6	7	8	9	10
11	12 * Pedersen 10:30-12:30	13 * Chiarellotto 11:00-13:00 * Pillonetto 10:30-12:30	14 * Chiarellotto 09:30-11:30 * Pedersen 10:30-12:30 * Brezinski 11:30-13:30	15 * Pillonetto 10:30-12:30 * Brezinski 11:30-13:30	16	17
18	19 * Pedersen 10:30-12:30	20 * Pillonetto 10:30-12:30 * Vargiolu 10:00-12:00	21 * Chiarellotto 09:30-11:30 * Pedersen 10:30-12:30 * Brezinski 11:30-13:30 * Abe 14:00-16:00	22 * Chiarellotto 09:30-11:30 * Pillonetto 10:30-12:30 * Brezinski 11:30-13:30 * Abe 14:00-16:00	23 * Vargiolu 10:00-12:00	24
25	26 * Pedersen 10:30-12:30	27 * Pillonetto 10:30-12:30 * Vargiolu 10:00-12:00	28 * Chiarellotto 09:30-11:30 * Pedersen 10:30-12:30 * Brezinski 11:30-13:30 * Abe 14:00-16:00	29 * Chiarellotto 09:30-11:30 * Pillonetto 10:30-12:30 * Brezinski 11:30-13:30 * Abe 14:00-16:00	30 * Vargiolu 10:00-12:00	31
1	2	3	4	5	6	7

novembre 2015

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
1	2 * Pedersen 10:30-12:30	3 * Pillonetto 10:30-12:30 * Vargiolu 10:00-12:00	4 * Chiarellotto 09:30-11:30 * Pedersen 10:30-12:30 * Brezinski 11:30-13:30 * Abe 14:00-16:00	5 * Chiarellotto 09:30-11:30 * Pillonetto 10:30-12:30 * Brezinski 11:30-13:30 * Abe 14:00-16:00	6 * Efthymiopoulos 10:30-12:30	7
8	9 * Pedersen 10:30-12:30	10 * Efthymiopoulos 09:00-11:00 * Chiarellotto 11:00-13:00 * Pillonetto 10:30-12:30	11 * Pedersen 10:30-12:30 * Chiarellotto 09:30-11:30 * Efthymiopoulos 14:30-16:30 * Brezinski 11:30-13:30	12 * Efthymiopoulos 09:30-11:30 * Pillonetto 10:30-12:30 * Brezinski 11:30-13:30	13	14
15	16	17 * Efthymiopoulos 09:00-11:00 * Chiarellotto 11:00-13:00 * Pillonetto 10:30-12:30	18 * Chiarellotto 09:30-11:30 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Brezinski 11:30-13:30	19 * Efthymiopoulos 09:30-11:30 * Pillonetto 10:30-12:30 * Brezinski 11:30-13:30	20 * Efthymiopoulos 10:30-12:30	21
22	23	24 * Pillonetto 10:30-12:30	25	26 * Pillonetto 10:30-12:30	27	28
29	30	1	2	3	4	5
6	7	8	9	10	11	12

dicembre 2015

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
29	30	1	2 * Seminario Dottorato (Matematica) - 14:30-16:00	3	4	5
6	7	8	9	10	11	12
13	14	15	16 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31	1	2
3	4	5	6	7	8	9

gennaio 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
27	28	29	30	31	1	2
3	4	5	6	7	8	9
10	11 * Burenkov 11:00-13:00	12 * Burenkov 11:00-13:00 * Favaro 14:30-16:30	13 * Zennaro 15:00-17:00 * Burenkov 11:00-13:00	14 * Zennaro 10:00-12:00 * Favaro 14:30-16:30	15	16
17	18 * Conforti 14:00-16:00 * Burenkov 11:00-13:00	19 * Burenkov 11:00-13:00 * Favaro 14:30-16:30	20 * Seminario Dottorato (Matematica) - 14:30-16:00 * Burenkov 11:00-13:00	21 * Zennaro 15:00-17:00 * Favaro 14:30-16:30	22 * Conforti 14:00-16:00 * Zennaro 10:00-12:00	23
24	25 * Conforti 11:00-13:00	26 * Zennaro 15:00-17:00 * Favaro 14:30-16:30 * Caravenna-Donadello 10:00-12:00	27 * Conforti 14:00-16:00 * Zennaro 10:00-12:00	28 * Favaro 14:30-16:30	29 * Conforti 11:00-13:00	30
31	1	2	3	4	5	6

febbraio 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
31	1	2 * Favaro 14:30-16:30 * Caravenna-Donadello 10:00-12:00	3 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Conforti 11:00-13:00	4 * Favaro 14:30-16:30	5	6
7	8	9	10 * Caravenna-Donadello 10:00-12:00	11	12	13
14	15	16 * Caravenna-Donadello 10:00-12:00	17 * Seminario Dottorato (Matematica) - 14:30-16:00	18	19	20
21	22 * Marcuzzi 10:30-12:30	23 * Caravenna-Donadello 10:00-12:00	24 * Marcuzzi 10:30-12:30	25	26 * Caravenna-Donadello 10:00-12:00	27
28	29 * Marcuzzi 10:30-12:30	1	2	3	4	5
6	7	8	9	10	11	12

marzo 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
28	29	1 * Caravenna-Donadello 10:00-12:00	2 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Marcuzzi 10:30-12:30 * Carnovale 11:30-13:30	3 * Carnovale 11:30-13:30	4	5
6	7 * Marcuzzi 10:30-12:30	8 * Caravenna-Donadello 10:00-12:00	9 * Marcuzzi 10:30-12:30 * Carnovale 11:30-13:30	10 * Carnovale 11:30-13:30	11	12
13	14 * Marcuzzi 10:30-12:30	15 * Caravenna-Donadello 10:00-12:00	16 * Seminario Dottorato (Matematica) - 14:30-16:00 * Marcuzzi 10:30-12:30 * Carnovale 11:30-13:30	17 * Carnovale 11:30-13:30	18	19
20	21 * Damm-Karow 16:30-18:30	22 * Caravenna-Donadello 10:00-12:00	23 * Damm-Karow 14:30-16:30 * Carnovale 11:30-13:30	24	25	26
27	28	29 * Caravenna-Donadello 10:00-12:00	30 * Damm-Karow 14:30-16:30 * Dey 10:30-12:30	31	1	2
3	4	5	6	7	8	9

aprile 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
27	28	29	30	31	1 * Damm-Karow 10:30-12:30	2
3	4 * Dey 10:30-12:30	5 * Cerf 13:00-15:00 * Caravenna-Donadello 10:00-12:00	6 * Damm-Karow 14:30-16:30 * Ruf 16:00-18:00 * Dey 10:30-12:30	7 * Cerf 11:00-13:00 * Ruf 14:30-16:30	8 * Ruf 16:00-18:00 * Damm-Karow 10:30-12:30	9
10	11 * Dey 10:30-12:30	12 * Ruf 14:30-16:30 * Cerf 11:00-13:00	13 * Carnovale 11:30-13:30 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Ruf 16:00-18:00 * Damm-Karow 14:30-16:30	14 * Ruf 14:30-16:30 * Carnovale 11:30-13:30	15 * Cerf 11:00-13:00 * Damm-Karow 10:30-12:30	16
17	18 * Dey 10:30-12:30 * Finesso 14:30-16:30	19 * Cerf 11:00-13:00	20 * Seminario Dottorato (Matematica) - 14:30-16:00 * Dey 10:30-12:30 * Carnovale 11:30-13:30 * Finesso 14:30-16:30	21 * Cerf 09:30-11:30 * Dai Pra-Fischer 15:00-17:00 * Carnovale 11:30-13:30	22	23
24	25 * Dey 10:30-12:30 * Finesso 14:30-16:30	26 * Dai Pra-Fischer 09:30-11:30	27 * Rovira Escofet 11:00-13:00 * Dey 10:30-12:30 * Carnovale 11:30-13:30 * Finesso 14:30-16:30	28 * Dai Pra-Fischer 15:00-17:00 * Carnovale 11:30-13:30	29	30
1	2	3	4	5	6	7

maggio 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
1	2 * Virili 11:30-13:30 * Dey 10:30-12:30 * Finesso 14:30-16:30	3 * Dai Pra-Fischer 09:30-11:30 * Virili 11:30-13:30	4 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Carnovale 11:30-13:30 * Finesso 14:30-16:30	5 * Dai Pra-Fischer 15:00-17:00 * Carnovale 11:30-13:30	6 * Dai Pra-Fischer 09:30-11:30 * Virili 11:30-13:30	7
8	9 * Virili 11:30-13:30 * Finesso 14:30-16:30	10	11 * Rovira Escofet 09:00-11:00 * Virili 14:30-16:30 * Carnovale 11:30-13:30 * Finesso 14:30-16:30	12 * Dai Pra-Fischer 14:30-16:30 * Carnovale 11:30-13:30	13 * Virili 11:30-13:30 * Dai Pra-Fischer 09:30-11:30	14
15	16 * Virili 11:30-13:30 * Finesso 14:30-16:30	17 * Virili 11:30-13:30	18 * Seminario Dottorato (Matematica) - 14:30-16:00 * Rovira Escofet 11:00-13:00 * Carnovale 11:30-13:30 * Finesso 14:30-16:30	19 * Dai Pra-Fischer 14:30-16:30 * Carnovale 11:30-13:30	20 * Virili 11:30-13:30 * Dai Pra-Fischer 09:30-11:30	21
22	23 * Virili 11:30-13:30 * Finesso 14:30-16:30	24	25 * Carnovale 11:30-13:30 * Finesso 14:30-16:30	26 * Dai Pra-Fischer 14:30-16:30	27 * Dai Pra-Fischer 09:30-11:30	28
29	30	31	1	2	3	4
5	6	7	8	9	10	11

giugno 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
29	30	31	1 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Rovira Escofet 08:30-10:30 * Ticozzi 10:30-12:30	2	3	4
5	6 * Ticozzi 10:30-12:30	7	8 * Rovira Escofet 09:00-11:00 * Ticozzi 10:30-12:30	9	10	11
12	13 * Ticozzi 10:30-12:30	**14**	15 * Seminario Dottorato (Matematica) - 14:30-16:00 * Ticozzi 10:30-12:30	16	17 * Schlogl 14:30-17.00 * Schlogl 10:00-12.30	18
19	20 * Ticozzi 10:30-12:30	21 * Schlogl 10:00-12.30 * Schlogl 14:30-17.00	22 * Ticozzi 10:30-12:30	23	24	25
26	27 * Ticozzi 10:30-12:30	28	29	30	1	2
3	4	5	6	7	8	9

luglio 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
26	27	28	29	30	1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22 * Mityushev 10:00-12:00	23
24	25	26	27	28	29	30
31	1	2	3	4	5	6

agosto 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
31	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1	2	3
4	5	6	7	8	9	10

settembre 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
28	29	30	31	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30 * Fine anno Dottorato	1
2	3	4	5	6	7	8