Applications of Canonical Perturbation Theory in Dynamical Astronomy

Prof. Christos Efthymiopoulos[1]

[1] Research Director at the
Research Center for Astronomy and Applied Mathematics, Academy of Athens
Email: cefthim@Academyofathens.gr

Timetable: 14 hrs. First lecture on November 6, 2015, 10:30, (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Examination and grading: Students will be assigned with small individual projects, aiming to develop their ability to make computations in canonical perturbation theory. A final written report on each project will have to be delivered upon the course's end. The evaluation will be based on the written reports.

SSD: MAT/07 - Mathematical Physics

Aim: The course aims to provide an application-oriented introduction to the techniques and tools of canonical perturbation theory. To this end, the course presents examples of concrete series computations allowing to draw information about the dynamics in some systems encountered in dynamical astronomy. In particular, the examples dealt with are:

1. the spin-orbit problem and the location of secondary resonances in the case of planets or satellites trapped in a spin-orbit resonance;
2. the secular dynamics in a fictitious planetary system composed of one star and two planets;
3. center (normally hyperbolic) manifold computations near the Lagrangian point L1 in the circular restricted three body problem (applying to the orbits of spacecrafts exploiting the so-called "space manifold dynamics");
4. adiabatic invariants and the motion of charged particles in magnetospheres.

All examples will be substantiated by computer-algebraic demonstrations during the lectures.

Course contents:

1. Introduction to the canonical formalism. Canonical transformations with Lie series. Hamiltonian normal forms. Small divisors. Non-resonant series. Resonances. Asymptotic behavior.
2. Simple examples of application: the time-modulated perturbed pendulum problem, Hamiltonian systems of two or three degrees of freedom with a convex integrable part. Estimates on chaos thresholds and diffusion.
3. Spin-orbit problem: formulation, 1:1 (Earth-Moon) and 3:2 (Sun-Mercury) resonances. Birkhoff normal form around the 1:1 resonance. Location of the 3:1 secondary resonance.
4. Secular dynamics: the Hamiltonian of the three-body problem in the Poincaré variables. Modified Delaunay variables. Disturbing function, expansion of. Averaging over short period terms. Determination of the secular solution.
5. Center manifold computations in the CRTBP: computation of the hyperbolic normal form. Reduction to the center manifold. Location and stability of the halo family of orbits.
6. Adiabatic invariants: magnetic bottle Hamiltonians. Normalization in the case of a nilpotent kernel. Computation of mirror frequencies. Resonances and the onset of global chaos.