Applied Functional Analysis and Machine Learning

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Timetable: 28 hrs (2 two-hours lectures per week): Classes on Tuesday and Thursday, 10:30 - 12:30. First lecture on Tuesday October 13th, 2015. Sala Riunioni 318 DEI/G 3-rd oor, via Gradenigo 6).

Course requirements: The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. The arithmetic of complex numbers and the basic properties of the complex exponential function. Some elementary set theory. A bit of linear algebra. All the necessary material can be found in W. Rudin's book Principles of Mathematical Analysis (3rd ed., McGraw-Hill, 1976). A summary of the relevant facts will be given in the first lecture.

Examination and grading: Homework assignments and final test.

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

Course contents:

1. *Review of some notions on metric spaces and Lebesgue integration*: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.
2. *Banach and Hilbert spaces*: Normed spaces and Banach spaces. Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces. Hilbert adjoint operator. Self-adjoint operators, unitary operators.
3. *Compact linear operators on normed spaces and their spectrum*: Spectral properties of bounded linear operators. Compact linear operators on normed spaces. Spectral properties of compact linear operators. Spectral properties of bounded self-adjoint operators, positive operators, operators defined by a kernel. Mercer Kernels and Mercer's theorem.
4. *Reproducing kernel Hilbert spaces, inverse problems and regularization theory*: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Basic concepts of convex analysis. Primal and dual formulation of loss functions. Regularization networks. Support vector regression and classification. Support vector classification.

References:

[1] W. Rudin. Real and Complex Analysis, McGraw Hill, 2006.
[2] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons, 1978.
[3] G. Wahba. Spline models for observational data. SIAM, 1990.
[4] C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006.
[5] R.T. Rockafellar. Convex analysis. Princeton University Press, 1996.