Topics in Numerical Probability with applications to advanced Quantitative Finance

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Timetable: 10 hrs, first lecture on June 17, 2016, 10:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements:
Standard probability theory and stochastic calculus

Examination and grading:
Project

SSD: INF/01 Computer Science, MAT06 Probability

Title:
Topics in Numerical Probability with applications to advanced Quantitative Finance

Course contents:
Session 1

Gram/Charlier Type A Series Expansions and an application to market-implied distributions and option pricing

If a probability distribution is sufficiently close to a normal distribution, its density can be approximated by a Gram/Charlier Series A expansion. In finance, this has been used to fit risk-neutral asset price distributions to prices observed on the market, ensuring an arbitrage-free interpolation of implied volatilities across exercise prices. We will discuss the option pricing formula in terms of the full (untruncated) series and consider a fitting algorithm, which ensures that a series truncated at a moment of arbitrary order represents a valid probability density. Generalising the Gram/Charlier Series A approach to the multiperiod, multivariate case, a model calibrated to standard option prices is developed, in which a large class of exotic payoffs can be priced in a single formula.

Outline:

1. Background and some useful lemmas 2. The Gram/Charlier expansion 3. Modelling asset price distributions 4. Extending the model to multiple assets and maturities 5. Further directions and applications

Session 2

Generic and object-oriented programming techniques for Monte Carlo simulation in C++

In many numerical probability applications, MC simulation is the easiest numerical method to implement, and for highdimensional problems it is often the most efficient. It is also a field of application where generic and object-oriented programming techniques assist in building a powerful toolbox. Following a philosophy that ideally every bit of functionality should be implemented once (only) in a well-designed library, I will discuss how building blocks can be created, representing the generic Monte Carlo algorithm, various control variate techniques, the Longstaff/Schwartz evaluation of optimal stopping problems, and quasi-random number generation. In addition to their applicability to more straightforward cases, these same building blocks can be combined, for example, to analyse complex financial instruments like Bermudan products which combine equity, interest rate and currency risk under any risk-neutral or real-world probability measure, or to combine control variate techniques with quasi-random number generation, in each case without the need to re-code the building blocks.

Outline:

1. Background & approach 2. Generic algorithm implementation 3. Extensions for variance reduction 4. Early exercise 5. Varying the building blocks Embarrassingly parallel? Parallel computing issues

Sessions 3 & 4 Fundamentals of Credit Risk Modelling for Counterparty Credit Risk Assessment and Valuation

This talk will cover the main concepts of counterparty credit risk in derivative financial instruments, i.e. expected exposure (EE), potential future exposure (PFE), credit valuation adjustment (CVA), and related values. It will focus on those aspects of mathematical modelling of credit risk needed to calculate counterparty credit risk as represented by these concepts, and discuss how CVA can be "marked-to-market" based on risk-neutral default probabilities implied by market credit spreads. In a simple, analytically tractable example, these concepts and calculations will be illustrated in a spreadsheet implementation. Outline: 1. Defining and modelling counterparty risk 2. Measuring counterparty risk 3. Obtaining implied default probabilities 4. Counterparty risk calculations