

# Selected Topics In Homological Algebra

Simone Virili<sup>1</sup>

<sup>1</sup> *Departamento de Matemáticas, Autonomous University of Barcelona, Spain*  
Email:

**Timetable:** 10 hrs. First lecture on May 2, 2016, 11:30, (dates already fixed, see the calendar)  
Torre Archimede, Room 2BC/30.

**Course requirements:** Some knowledge of category theory and of Abelian categories. Basic facts from ring and module theory will be of great help.

**Examination and grading:** Lists of exercises will be assigned weekly. There will be also a final oral examination.

**SSD:** MAT/03

**Aim:** The aim of this lecture series is to give an overview of the classical theory of localizations of Grothendieck categories (due to Grothendieck, Gabriel, Popescu, and others), and to underline similarities and differences with the theory of localization of derived categories (giving an overview of results of Neeman, Krause, Balmer, Stevenson, Rickard and others). To do so, we will need to develop along the way some of the basic techniques of triangulated and derived categories.

## Course contents:

1. Grothendieck Categories:
  - (a) Grothendieck categories, definitions and basic facts;
  - (b) torsion theories and localization of Grothendieck categories;
  - (c) the Gabriel filtration and the Gabriel dimension;
  - (d) Gabriel's classification of hereditary torsion theories.
2. Derived and triangulated categories:
  - (a) the axioms of a triangulated category;
  - (b) our main example: the derived category of a Grothendieck category;
  - (c) derived functors;
  - (d) homotopy (co)limits and resolution of unbounded complexes.
3. Support theory for triangulated categories:
  - (a) tensor-triangulated categories;
  - (b) localization of triangulated categories;
  - (c) the Balmer spectrum of triangulated ideals;
  - (d) the classification of localizing subcategories in a triangulated category.

## References:

1. P. Balmer, The spectrum of prime ideals in tensor triangulated categories
2. M. Bkstedt, A. Neeman, Homotopy limits in triangulated categories

3. P. Gabriel, Des catégories abéliennes
4. H. Krause, Localization theory for triangulated categories
5. A. Neeman, Triangulated categories
6. A. Neeman, The chromatic tower for  $D(\mathbb{R})$
7. N. Popescu, Abelian categories with applications to rings and modules
8. B. Stenström, Rings of Quotients, An Introduction to Methods of Ring Theory
9. G. Stevenson, Support theory via actions of triangulated subcategories