

Lie Algebras

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Timetable: 20-24 hours. Lectures on Monday 09:30-11:15 and Thursday 11:30-13:15. First lecture on April 3rd, 2017, Torre Archimede, Room 2AB/40.

Course requirements: Basic notions of linear algebra

Examination and grading: exercises

SSD: MAT/02

Aim: This course provides an introduction to Lie algebras and aims at presenting the classification of complex simple Lie algebras.

Course contents:

1. Basic notions. The adjoint representation and its subrepresentations. Derived subalgebra. Solvable and nilpotent Lie algebras. Nilpotent elements are ad-nilpotent.
2. Engel's theorem and Lie's theorem.
3. Irreducible representations of solvable Lie algebras. Schur's lemma.
4. Irreducible representations of $\mathfrak{sl}(2, \mathbb{C})$. Uniqueness of the Jordan decomposition in $\text{End}(V)$
5. Killing form. Cartan's solvability criterion.
6. Cartan's semisimplicity criterion. Trace forms and Casimir element. Weyl's theorem.
7. Cartan subalgebras. Abstract Jordan decomposition.
8. The root space decomposition. \mathfrak{sl}_2 -triples.
9. Reductive Lie algebras. Root strings. Euclidean structure on the real span of roots.
10. Root systems and Weyl group.
11. Strategy for the classification of classical Lie algebras. Simple Lie algebras have irreducible root systems and viceversa.
12. Classical Lie algebras are simple (up to two cases).
13. Serre's theorem. Uniqueness of the semisimple Lie algebra associated with a root system. Uniqueness of the root system associated with a Lie algebra.