Doctoral Course in Mathematical Sciences
Department of Mathematics
University of Padova

Doctoral Course in Mathematical Sciences

Catalogue of the courses 2017

Updated June 19, 2017

INTRODUCTION

The courses offered, for the year 2017, to the Graduate Students in Mathematical Sciences include courses taught by internationally recognized external researchers, who have accepted our invitation; such courses will not necessarily be offered again in the future years. Considering the wide impact of the content of theses courses, we emphasize the important for all graduate students to follow them.
The Faculty of the Graduate School could cancel courses with an excessively low number of registered students.

Also next year, beside the courses that our Doctoral Course directly offers, we have selected some courses of the Graduate School in Information Engineering of the University of Padova that we consider relevant also for our Course.

REQUIREMENTS FOR GRADUATE STUDENTS

With the advice of some Faculty member, all students are required to select some courses, either because they are linked with the curriculum of their present or planned research, or just to improve their knowledge of specific subject.

This year, considering the fact that courses may vary in duration, we have decided to indicate a mandatory minimum numbers of hour.
Therefore, students are required, within the **first two years (a half of the requirements within the first year)**, to follow and **pass the exam** of

- **at least 2 among the courses offered by the PhD Programme called "Courses of the School";**
- other courses in the catalogue, <u>in addition to the two above</u>, with a total commitment **of additional 64 hours**.

Students are encouraged to register for other courses; although to sit for the exam is not required for these courses, it is strongly advised. In all cases, students must participate with regularity to the activities of the courses they are registered to. At the end of the course the teacher will inform the Coordinators of the Curricula on the activities of the course and of the registered students.

<u>Institutional courses for Master of Science in “Mathematics”</u>

Students have the possibility to attend, with acquisition of credits, the courses of the Master of Science in Mathematics.
The interest for these courses must be indicated by the Supervisor or a tutor. The Council will assign the number of hours that will be computed within the mandatory 64 hours.

<u>Courses attended in other Universities</u>

Students are allowed to take Ph.D. courses offered by PhD Programmes of other Universities. Acquisition of credits will be subject to approval of the Council.

HOW TO REGISTER TO COURSES

The online registration to courses has changed from last years, and allows students both to register and to cancel.
The registration is required for the attendance to all courses, independently of the intention to sit for the exam. The list of the courses can be found in the website of the Doctoral Course http://dottorato.math.unipd.it/ at the link Courses Registration (or directly at the address http://dottorato.math.unipd.it/registration/), filling the **online registration form** with all required data, and validating with the command "Register".
To acknowledge the registration, an email message will be sent to the address indicated in the registration form; this email message must be saved, since it is necessary for possible cancellation.

Registration for a course implies the agreement of the applicant to the participation.
Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except those that begin in October and November) using the link indicated in the email message of acknowledgment.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in the catalog, although part of activities in the Graduate School in Mathematics and thus offered to its students, are also open to all students, graduate students and researchers of all Graduate Schools and other universities.
For reasons of organization, external participants are required to **indicate their wish to participate at least two months before the beginning of the course for courses taking place from January 2017 and as soon as possible for courses that take place until December 2016, following the procedure described in the preceding paragraph.**
Possible **cancellation** to courses must also be notified.

Courses of the School

Courses of the “Computational Mathematics” area

Courses of the “Mathematics” area

3. Prof. Andrea D'Agnolo
   Topology 2 **M-3**

4. Prof. Alexey Karapetyants
   Bergman spaces of analytic functions: from classical results to the new settings and ideas involving nonstandard behavior of function and symbols of operators **M-4**

5. Prof. Nicola Mazzari
   De Rham Cohomology **M-7**

6. Prof. Gabriella Pinzari
   The Kolmogorov-Arnold-Moser theorem and its applications to the N-body problem **M-8**

7. Prof. Luigi Salce
   A soft introduction to algebraic entropy **M-10**

8. Prof. Alfonso Sorrentino
   Weak KAM and Aubry-Mather Theory **M-11**

Courses in collaboration with the Doctoral School on "Information Engineering"

1. Proff. F. De Terán, Michael Karow
   Applied Linear Algebra **DEI-1**

2. Prof. Giorgio Maria Di Nunzio
   Bayesian Machine Learning **DEI-2**

3. Prof. Lorenzo Finesso
   Statistical methods **DEI-4**

4. Prof. J. Hauser
   Optimization and Optimal Control **DEI-5**

5. Prof. Fabio Marcuzzi
   Computational Inverse Problems **DEI-6**

6. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-7**

7. Proff. Rodolphe Sepulchre, Fulvio Forn
   Local Methods for Nonlinear Systems and Control **DEI-9**

Courses of the School

Partial Differential Equations and Dynamic Games

Prof. Martino Bardi[1], Prof. Yves Achdou[2]

[1]University of Padova
Department of Mathematics
Email: bardi@math.unipd.it

[2]University of Paris-Diderot
Laboratoire Jacques-Louis Lions and UFR de Maths
Email: achdou@ljll.univ-paris-diderot.fr

Timetable: about 22-24 hrs, first part (about 16 hours) by Bardi, second part (6-8 hours) by Achdou.
(Part one) First lecture on November 18th, 2016, 11:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.
(Part two) First lecture on February 13, 2017, 11:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30 (Room 2AB/45 for the lecture of February 16, 2017).

Course requirements: Basic knowledge of PDEs and stochastic processes.

Examination and grading: the exam will be tailored on the students who will attend the course.

SSD: MAT/05 (Math. Analysis) and MAT/06 (Probability)

Aim: to present some classical connections among PDEs and diffusion processes, optimal control and differential games, and to provide an introduction to the recent theory of Mean-Field Games.

Course contents:

- Motivations: deterministic and stochastic controlled dynamical systems, optimisation problems for one and for many players (Nash equilibria).
- Recall of basic stochastic calculus and partial differential operators associated to diffusion processes.
- Verification theorems for optimal control problems and differential games, Hamilton-Jacobi-Bellman and Isaacs equations. Examples: Linear-Quadratic problems, Merton's model of optimal portfolio selection, …
- The Kolmogorov-Fokker-Planck equation.
- Dynamic Programming and viscosity solutions of Hamilton-Jacobi-Bellman equations.
- The Mean-Field Game system of PDEs: heuristic derivation.
- Connection of the Mean-Field Game equations with the large population limit of N-player games.
- Selected topics in Mean-Field Games (prof. Yves Achdou).

Introduction to GAGA type theorems

Prof. Bruno Chiarellotto[1]

[1] *University of Padova*
Department of Mathematics
Email: chiarbru@math.unipd.it

Timetable: 20 hrs. First lecture on November 16th, 2016, 11:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basis Commutive algebra. Basic Algebraic Topology. Basic Algebraic geometry and differential geometry.

Examination and grading: the exam will be oral and tailored on the basis of the student's attitude.

SSD: Mat02/03/05/07

Aim: We will introduce some basis tools from classical algebraic geometry to complex analytic spaces. We will try to tailor the class on the basis of the students' attitude.

Course contents: A set characterized as common zeroes of some polynomial in several variables over the complex numbers admits naturally two different topology. One coming from the **A**lgebraic **G**eometry (the Zariski's one) and one coming from the **A**nalytic **G**eometry (the trascendental one given by usual Archimedean absolute value). Can we calculate the invariants of the analytic setting via only algebraic setting? And viceversa? This kind of problems is indicated as "GAGA" theorems. We will try to introduce the setting and proving some results

Covering trees: alea and application

Prof. Alexandre Gaudillière[1]

[1]CNRS, Institute de Mathematiques de Marseille
Email: alexandre.gaudilliere@math.cnrs.fr

Timetable: 24 hours. First lecture on June 5, 2017, 11:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: The course assumes the knowledge of basic probability, including the elementary theory of Markov Chains. If necessary, 1-2 introductory lectures will be given before the course.

Examination and grading: Oral exam.

SSD: MAT/06 Probability and Mathematical Statistics

Aim:
Covering trees of connected graphs are basic tools in operation research and communication protocols. As they are seen as random object, they reveal a number of connections between seemingly distant areas of mathematics. Starting with Kirchoff work, they have been use to build connections between the theory of electrical networks, algebra and combinatorics: one could for instance obtain the intensity of the electrical current in a network through the computation of determinants related to the counting of covering trees for the network. They have the become key tools in the study of the so-called *determinantal processes*, whose properties can be expressed in terms of the rendom walk on the underlying graph. The resulting formulas are also useful for the spectral analysis of graphs, and for solving models in Statistical Mechanics.

Course contents:
The course will begin with a detailed analysis of the connections mentioned above, in the case of finite graphs. We will then make extensions to random graphs and infinite graphs, working either directly in the infinite graphs of by considering limit of sequences of finite graphs. We will see connections with the associated random walks, some recent results and open problems. The last part of the course will be devoted to applications to multiscale analysis of graphs and markov processes, and to problems emerging in signal processing.

Courses of the “Computational Mathematics” area

New trends in Numerical Analysis and Scientific Computing

Prof. Claude Brezinski[1]

[1] *Laboratoire Paul Painlevé, UMR CNRS 8524*
Université des Sciences et Technologies de Lille, France
Email: Claude.Brezinski@univ-lille1.fr

Timetable: 18-20 hours. March-May 2017, Torre Archimede,

Course requirements: No special requirement is needed for this course. Only some fundamental knowledge of numerical analysis, but it could be acquired simultaneously with the lectures.

Examination and grading: Grading is based on homeworks or a written examination or both.

SSD: MAT/08 Numerical Analysis

Aim: The aim of the lectures is to introduce PhD students to some recent research subjects in numerical analysis (especially those related to approximation and numerical linear algebra) and to provide them the theoretical basis for their understanding. Applications will also be discussed. These lectures are intended to students and researchers in pure and applied mathematics, in numerical analysis, and in scientific computing.

Course contents: The various topics developed at different levels, will be

1. Formal orthogonal polynomials
   (a) Definition
   (b) Algebraic properties
   (c) Recurrence relation
   (d) Adjacent Families
2. Padé approximation
   (a) Definition and algebraic properties
   (b) Padé-type approximants
   (c) Connection to formal orthogonal polynomials
   (d) Recursive computation
   (e) Connection to continued fractions
   (f) Some elements of convergence theory
   (g) Applications
3. Krylov subspace methods
   (a) Definition
   (b) Lanczos method
   (c) Recurrence relations
   (d) Implementation
4. Extrapolation methods
   (a) Sequence transformations and convergence acceleration
   (b) What is an extrapolation method?

(c) Various extrapolation methods
(d) Vector sequence transformations
(e) Applications
    i. Treatment of the Gibbs phenomenon
    ii. Web search
    iii. Estimation of the error for linear systems
    iv. Regularization of linear systems
    v. Estimation of the trace of matrix powers
    vi. Acceleration of Kaczmarz method
    vii. Fixed point iterations
    viii. Computation of matrix functions

References

[1] Lecture notes provided to the students following the courses.

[2] C. Brezinski, *Padé–Type Approximation and General Orthogonal Polynomials*, ISNM, vol. 50, Birkhäuser–Verlag, Basel, 1980.

[3] C. Brezinski, M. Redivo-Zaglia, *Extrapolation Methods. Theory and Practice*, North–Holland, Amsterdam, 1991. C. Brezinski, *Biorthogonality and its Applications to Numerical Analysis*, Marcel Dekker, New York, 1992.

[4] C. Brezinski, *Projection Methods for Systems of Equations*, North-Holland, Amsterdam, 1997.

[5] C. Brezinski, *Computational Aspects of Linear Control*, Kluwer, Dordrecht, 2002.

Integer Programming and Lattices

Prof. M. Conforti

Università di Padova
Dipartimento di Matematica
Email: conforti@math.unipd.it

Timetable: 12 hrs. First lecture on November 21st, 2016, 14:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30 and Meeting Room IV and VII floor.

Course requirements: basic Linear Algebra.

Examination and grading: to be discussed with students.

SSD: MAT/09 Operations Research.

Aim: Introducing Lattice Theory and its connections with discrete optimization, in particular Integer Programming.

Course contents: Lattice Theory is pervasive in Integer Programming. The following topics will be discussed:

- Fundamental concepts in Lattice Theory
- Minkowski's convex body theorem
- Orthogonality defect and the LLL algorithm
- Shortest vector problem
- Löwner-John ellipsoids and lattice width
- Integer Programming in fixed dimension

Growth optimality and recent applications to probability

Prof. Constantinos Kardaras

London School of Economics and Political Sciences, UK
Email: k.kardaras@lse.ac.uk

Timetable: 10 hrs. First lecture on April 3rd, 2017, 16:30 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Probability and Stochastic Calculus

Examination and grading: Written examination (short essay or exercises)

SSD: MAT/06 e SECS-S/06

Aim: This course aims at providing an overview on recent developments in the Mathematical Finance research field, having as a central idea in mind the role of the growth optimal portfolio.

Course contents:
These lectures will touch upon recent developments in the applied probability, and more precisely mathematical finance. The vessel that is used to connect different topics is that of growth optimality, a notion that has proved extremely fruitful. A representative collection of what will be covered are applications in arbitrage theory, constrained optimisation, filtration enlargement, and semimartingale theory. Other areas of application, such as economics, robust optimisation, and even functional analysis, will be also discussed (if time permits).

Modern Aspects of Constructive Function Theory

Prof. András Kroó[1]

[1] Department of Mathematical Analysis
Budapest University of Technology and Economics

Alfréd Rényi Institute of Mathematics
Budapest, Hungary
Email: kroo@renyi.hu

Timetable: 16 hrs. Torre Archimede
Date: January 10, 11, 12, 13, 17, 18, 19, 20, 2017
Rooms and time: the lectures of Tuesday 10 and 17 are in room 2BC30, 10:30-12:00; the lectures of Friday 13 and 20 are in room 2BC30, 9:30-11:00.
The other lectures as usual room 2AB45, 11:30-13:00.

Course requirements: Attendance of lectures and successful final oral test

Examination and grading: oral exam at the end of course

SSD: MAT08, Subject: Approximation Theory

Aim: The main goal of the course is to introduce students to the main topics and methods of the Approximation Theory. The learning outcomes of the course: By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well.

Course contents:

1. Stone-Weierstrass theorem, positive linear operators. Korovkin theorem.
2. Best approximation in various Banach spaces. Haar spaces and Chebyshev Theorem.
3. Classical polynomial inequalities (Bernstein, Markov, Remez inequalities).
4. Direct and converse theorems of best approximation (Favard and Jackson Theorems).
5. Approximation by linear operators (Fourier and Fejér operators, Bernstein Polynomials).
6. Lacunary polynomial approximation, incomplete polynomials, Müntz theorem.
7. Bernstein-Markov type inequalities for multivariate polynomials on convex and star like domains in uniform and integral norms.
8. Markov type inequalities for homogeneous polynomials on convex bodies.
9. Tangential Bernstein-Markov type inequalities.
10. Remez type inequalities for multivariate polynomials on star like domains and convex bodies and their application.
11. Admissible and optimal meshes for multivariate polynomials.
12. Some contemporary aspects of density for multivariate functions. Approximation by ridge functions and incomplete polynomials in several variables. Approximation by homogeneous polynomials on the boundary of convex domains. Approximation of convex bodies by convex algebraic level surfaces

Reference: R. DeVore and G. Lorentz, Constructive Approximation, Springer, 1991.

Option Pricing: from Monte Carlo to Quantization

Giorgia Callegaro[1], Lucio Fiorin[2], Daniele Marazzina[3]

[1] *Università di Padova*
Email: gcallega@math.unipd.it

[2] *Università di Padova*
Email: fiorin@math.unipd.it

[2] *Politecnico di Milano*
Email: daniele.marazzina@polimi.it

Timetable: 24 hours, Spring 2017, at Politecnico di Milano.

Course requirements: The course assumes the knowledge of basic probability.

Examination and grading: Code development on a specific project.

SSD: SECS-S/06

Aim:
Monte Carlo methods are extensively used in finance to value and analyze complex instruments, portfolios and investments by simulating the various sources of uncertainty affecting their value. The advantage of Monte Carlo methods over other techniques increases as the dimensions (sources of uncertainty) of the problem increase. However, it is well known that the disadvantage of Monte Carlo methods is the slow convergence, and thus the high computational cost of the algorithms. Quantization is a way to approximate a random vector or a stochastic process using a nearest neighbour projection on a finite codebook. The birth of quantization dates back to the 1950s, when in the Bell laboratories ad hoc signal discretization procedures were developed for signal transmission. In the last years, Quantization has been deeply considered in numerical probability, especially for solving problems arising in mathematical finance, presenting itself as a de facto alternative to Monte Carlo methods. The purpose of the course is to highlight the characteristics of the two methodologies and to deeply analyze (and implement) their applications in financial context, mainly option pricing.

Course contents:

Pre-course (if necessary/required): Matlab introduction

First part: Monte Carlo (MC) Methods (Daniele Marazzina)

- Introduction to financial derivatives and to financial stochastic modelling
- MC methods and variance reduction (control variates and antithetic sampling)
- MC codes: exact schemes, Euler and Milstein schemse, QE scheme for Heston
- MC for jump processes

Second part: Quantization (Giorgia Callegaro)

- Introduction to quantization
- Quantization of a random variable
- Marginal Recursive quantization
- Applications of Marginal Recursive Quantization: barrier options and exotic option pricing

Third part: Implementationof quantization methods (Lucio Fiorin)

- 1D setting: Black Scholes, CEV, pricing of European options, barrier and American options
- 2D case: Heston, pricing of European and American options

References: Graf, Siegfried, and Harald Luschgy. Foundations of quantization for probability distributions. Springer, 2000.

Selected topics from graph theory

Prof. Christoph Mass

Hamburg University of Applied Sciences
Email: christoph.maas@haw-hamburg.de

Timetable: 8 hrs. Lectures on May 9 and 11, 2017, 13:30-17:30 , Torre Archimede, Room 2BC/30.

Course requirements: Basic knowledge in graph theory and discrete optimization (in Italian)

Examination and grading: To be discussed with the participants.

SSD: MAT/02/06/09

Aim:

- adding topics to the questions discussed in the courses “matematica discreta” and “ottimizzazione discreta”,
- addressing some of the prerequisites for the course “tree covering: alea and applications”

Course contents:

- English terminology in graph theory
- Extremal graph theory
- Algorithmic graph theory
- Random walks and Markov chains
- Eigenvalues of graphs

Topics in spectral theory for network analysis

Dott. Francesco Tudisco[1]

[1] *University of Padova*
Department of Mathematics
Email: tudisco.francesco@gmail.com

Timetable: 12 hrs. First lecture on November 8th, 2016, 11:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Background in numerical linear algebra, numerical and mathematical analysis.

Examination and grading: Oral presentation or written essay

SSD: MAT/08 Numerical Analysis; INF/01 ComputerScience; MAT/05 Mathematical Analysis

Aim: Provide an introduction to some fundamental topics of spectral theory for graph analysis, addressing some classical and some state-of-the-art models and techniques.

Course contents: Many mathematical models and numerical methods for handling network problems are based on spectral theory of linear operators. However, more recently, the introduction of nonlinear operators, the use of matrix functions, and the associated spectral theories has allowed for more general, accurate and efficient models and techniques.

The course will introduce to modern spectral-oriented network analysis by touching the following topics:

- Eigenvector centrality and centrality based on matrix functions
- Centrality in higher order networks (time-varying, multilayer, hypergraphs)
- Graph Laplacian and spectral partitioning
- p-Laplacian and nonlinear spectral clustering
- Nonlinear power method

References:

1. Estrada, Ernesto and Knight, Philip, *A first course in network theory*, 2015, Oxford University Press, USA
2. Gallier, Jean, *Spectral Theory of Unsigned and Signed Graphs. Applications to Graph Clustering: a Survey*, arXiv:1601.04692, 2016.

Topics in Stochastic Analysis

Prof. Tiziano Vargiolu[1]

[1] *Università di Padova*
Dipartimento di Matematica Pura ed Applicata
Email: vargiolu@math.unipd.it

Calendario: 10 hrs. First lecture on October 13, 2016, 14:30 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Prerequisiti: A previous knowledge of the basics of continuous time stochastic analysis with standard Brownian motion, i.e. stochastic integrals, Itô formula and stochastic differential equations, as given for example in the master course ”Analisi Stocastica”.

Tipologia di esame: Seminar

SSD: MAT/06

Programma: The program will be fixed with the audience according to its interests. Some examples could be:

- continuous time stochastic control;
- Levy processes;
- numerical methods;
- stochastic control.

Numerical methods for Ordinary Differential Equations

Prof. Marino Zennaro[1]

[1] *University of Trieste*
Department of Mathematics and Geosciences
Email: zennaro@units.it

Timetable: 12 hrs. Course cancelled.

Course requirements: it is advisable to have attended a basic course in Numerical Analysis.

Examination and grading: Written exam.

SSD: MAT/08 Numerical Analysis

Aim: We present basic numerical methods for initial value problems in ordinary differential equations and we analyse their convergence properties.

Course contents:
Existence and uniqueness of the solution and continuous dependence on the data for the initial value problem $y'(x) = f(x, y(x)),\ y(x_0) = y_0$.
Classical Lipschitz constant and right hand side Lipschitz constant.
General one-step methods; explicit and implicit Runge-Kutta methods.
Definition of local truncation and discretization error for one-step methods and definition of consistency of order p. Convergence theorem with order p for one-step methods. Order conditions for Runge-Kutta methods. Order barriers for explicit and implicit methods.
Variable stepsize implementation. Embedded pairs of methods of Runge-Kutta-Fehlberg type.

References:

- E. Hairer, S.P. Norsett, G. Wanner: Solving Ordinary Differential Equations I, Nonstiff Problems, Springer-Verlag, Berlin, 1993
- J.C. Butcher: Numerical methods for ordinary differential equations. Second edition, John Wiley & Sons, Ltd., Chichester, 2008
- Lecture notes

Courses of the “Mathematics” area

Lie Algebras

Prof. Giovanna Carnovale[1]

[1] *Università di Padova, Dipartimento di Matematica*
Email: carnoval@math.unipd.it

Timetable: 20-24 hours. Lectures on Monday 09:30-11:15 and Thursday 11:30-13:15. First lecture on April 3rd, 2017, Torre Archimede, Room 2AB/40.

Course requirements: Basic notions of linear algebra

Examination and grading: exercises

SSD: MAT/02

Aim: This course provides an introduction to Lie algebras and aims at presenting the classification of complex simple Lie algebras.

Course contents:

1. Basic notions. The adjoint representation and its subrepresentations. Derived subalgebra. Solvable and nilpotent Lie algebras. Nilpotent elements are ad-nilpotent.
2. Engel's theorem and Lie's theorem.
3. Irreducible representations of solvable Lie algebras. Schur's lemma.
4. Irreducible representations of sl(2,C). Uniqueness of the Jordan decomposition in End(V)
5. Killing form. Cartan's solvability criterion.
6. Cartan's semisimplicity criterion. Trace forms and Casimir element. Weyl's theorem.
7. Cartan subalgebras. Abstract Jordan decomposition.
8. The root space decomposition. sl2-triples.
9. Reductive Lie algebras. Root strings. Euclidean structure on the real span of roots.
10. Root systems and Weyl group.
11. Strategy for the classification of classical Lie algebras. Simple Lie algebras have irreducible root systems and viceversa.
12. Classical Lie algebras are simple (up to two cases).
13. Serre's theorem. Uniqueness of the semisimple Lie algebra associated with a root system. Uniqueness of the root system associated with a Lie algebra.

Representation Theory of Groups

Prof. Giovanna Carnovale[1]

[1] Università di Padova, Dipartimento di Matematica
Email: carnoval@math.unipd.it

Timetable: 14 hours. March, 2017, Torre Archimede,

Course requirements: Basic notions of linear algebra and of group theory

Examination and grading: exercises

SSD: MAT/02

Aim: This course provides an introduction to the representation theory of groups, with focus on character theory for complex representations of finite groups.

Course contents:

1. Basic notions of representation theory: representations, irreducible representations, completely reducible representations, indecomposable representations.
2. Tensor products, exterior and symmetric powers, duals, representiation structure on Hom spaces. Schur's lemma.
3. Characters and their main properties. Orthogonality relations. Isotypical components. Decomposition of the regular representation.
4. Complex irreducible characters are an orthonormal basis for the space of central functions.
5. Construction of irreducible representations for abelian groups. How to enumerate complex 1-dimensional representations in a finite group. Induced representations and their character.
6. Frobenius reciprocity. Algebraic integers. Dimension of an irreducible representation.
7. Frobenius-Schur indicator. Enumerating involutions in a finite group. Compact groups and their representation theory.

Topology 2

Andrea D'Agnolo[1]

[1] Universitá di Padova
Dipartimento di Matematica
Email: dagnolo@math.unipd.it

Timetable:
for information regarding the timetable of the classes please contact prof. Andrea D'Agnolo (dagnolo@math.unipd.it) before October 15th.

Course requirements:

Examination and grading:

SSD: MAT/03-MAT/05

Aim: see http://tiny.cc/topologia

Course contents: Algebraic Topology is usually approached via the study of the fundamental group and of homology, defined using chain complexes, whereas, here, the accent is put on the language of categories and sheaves, with particular attention to locally constant sheaves.

Sheaves on topological spaces were invented by Jean Leray as a tool to deduce global properties from local ones. This tool turned out to be extremely powerful, and applies to many areas of Mathematics, from Algebraic Geometry to Quantum Field Theory.

On a topological space, the functor associating to a sheaf the space of its global sections is left exact, but not right exact in general. The derived functors are cohomology groups that encode the obstructions to pass from local to global. The cohomology groups of the constant sheaf are topological (and even homotopical) invariants of the space, and we shall explain how to calculate them in various situations.

Bergman spaces of analytic functions: from classical results to the new settings and ideas involving nonstandard behavior of function and symbols of operators.

Prof. Alexey Karapetyants[1]

[1] *Southern Federal University and Don State Technical University, Russia*
Email: karapetyants@gmail.com

Timetable: 10 hours. First lecture on February 27, 2017, 10:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30

Course requirements: Knowledge of basic university facts on functions of real and complex variables, basic properties of Banach and Hilbert spaces, basic properties of linear operators on Banach and Hilbert spaces. Everything on the level of understanding basic university courses which include these subjects.

Examination and grading: During the course I will ask some questions for homework activity. Those who will complete these homework questions will get score up to 50% of the total. The final test (quiz) will be given to the students, and on the base of this test a student may have also up to 50% of the total score. Those students who will consider their score lower than they've expected will be provided with an opportunity to defend their answers with no restrictions on time after the last lecture.

SSD: MAT/05

Aim: The aim of the course is twofold. First, to provide the students with classical objects and classical (textbook) instruments on study of Bergman spaces and operators in these spaces. Second, to provide with a knowledge of further development of the study of Bergman type spaces and operators in nonstandard situations, where the symbols of operators or the functions from these spaces admit so-called nonstandard behavior in a neighborhood of the boundary.

Description of the Course:
Agenda (a short review of planned research)
This short lecture course is aimed to introduce the students with basic knowledge of classical Bergman spaces and Toeplitz operators, and to provide the motivation and recent results for developing of the theory of analytic Bergman type spaces of functions with nonstandard (non-classical) boundary behavior. The course is based on classical textbooks for Bergman spaces and operator theory, papers by the author in collaboration with N.Vasilevsky and S.Grudsky on Toeplitz operators with badly behaved special symbols, and papers by the author with S.Samko on some new mixed norm Bergman type spaces of functions with nonstandard growth near the boundary.

Work plan

This work plan is for either five lectures each consisting of two hours or for ten one-hour lectures.

1. Basic facts on classical Bergman spaces and Toeplitz type operators (4 hours)
   This part is aimed to provide background information for Bergman type spaces and classical Toeplitz operators. It includes the discussion of classical Bergman spaces and Toeplitz type operators with nice symbols, the Bergman kernel function, point evaluation functional, orthonormal bases, boundedness of Bergman projection, the Berezin transform of the Toeplitz operator and its connection with properties of Toeplitz operator. These basic results will be followed with an outline of some advanced results on special Toeplitz operators with unbounded and badly behaved symbols, which also serves as a motivation for further development of the theory in the direction of mixed norm and nonstandard growth space settings.
2. The definition and basic properties of new Bergman type mixed norm spaces: general approach (2 hours)
   This two hours lecture is aimed to providing a new general approach to the definition of mixed norm Bergman space on the unit disc in a general settings. We discuss the problem of boundedness of Bergman projection in this general setting, the meaning of distributional Fourier coefficients used in the definition of the spaces, completeness of these new spaces, and other basic facts.
3. Concrete new Bergman type mixed norm spaces (4 hours)
   We start with the definitions and basic properties of particular spaces of functions whose norms are used in the definition of the new Bergman type mixed norm spaces: variable Lebesgue spaces, Orlich and Morrey type spaces. Further we discuss each particular case (Bergman-variable Lebesgue, Bergman-Orlich, Bergman-Morrey) separately proving the boundedness of the Bergman projection, characterizing the function in these spaces and discussing the boundary behavior of functions in such spaces. At the conclusion we propose some open questions and provide with a short resume of future plans for research in this new direction of study of spaces and operators.

References for parts 1 and 2:

[1] Hedenmalm H., Korenblum B., Zhu K. Theory of Bergman spaces. New York: Springer Verlag, Inc.; 2000.

[2] Duren P., Schuster A. Bergman spaces. Vol. 100. Providence (RI): Mathematical Surveys and Monographs; 2004.

[3] Zhu K. Spaces of holomorphic functions in the unit ball. Graduate texts in Mathematics, Springer 2004.

[4] Zhu K. Operator theory in function spaces. Mathematical Surveys and Monographs, Vol. 138; 2007.

[5] Vasilevski N.L. Commutative Algebras of Toeplitz Operators on the Bergman Space. Vol.185: Operator Theory: Advances and Applications; 2008.

[6] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Toeplitz operators on the unit ball in $\mathbb{C}^n$ with radial symbols. J. Operator Theory. 2003; 49:325-346.

[7] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Dynamics of properties of Toeplitz operators with radial symbols. Integral Equations Operator Theory. 2004; 50:217-253.

[8] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Dynamics of properties of Toeplitz operators in the weighted Bergman spaces. Siberian Electronic Mathematical Reports. 2006; 3:362-383.

[9] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Dynamics of properties of Toeplitz operators on the upper half plane: Hyperbolic case. Bol. Soc. Mat. Mexicana. (2004); (3) 10:119-138.

[10] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Dynamics of properties of Toeplitz operators on the upper half plane: Parabolic case. J. Operator Theory. (2004); 52:185-214.

References for part 3:

[11] Diening L., Harjulehto P., Hasto P., Ruzicka M. Lebesgue and Sobolev spaces with variable exponents. Vol.2017. Springer-Verlag, Heidelberg: Lecture Notes in Mathematics; 2011.

[12] Cruz-Uribe D.,Fiorenza A. Variable Lebesgue Spaces. Foundations and Harmonic Analysis. Springer Basel: Applied and Numerical Harmonic Analysis; 2013.

[13] Kokilashvili V., Meskhi A., Rafeiro H., Samko S. Integral Operators in Non-Standard Function Spaces. Vol. I: Variable Exponent Lebesgue and Amalgam Spaces. Birkhauser Basel: Operator Theory: Advances and Applications; 2016.

[14] Kokilashvili V., Meskhi A., Rafeiro H., Samko S. Integral Operators in Non-Standard Function Spaces. Vol. II: Variable Exponent Holder, Morrey–Campanato and Grand Spaces. Birkhauser Basel: Operator Theory: Advances and Applications; 2016.

[15] Karapetyants A., Samko S. Mixed norm variable exponent Bergman space on the unit disc. Complex Variables and Elliptic Equations. Vol. 61, Issue 8, 2016; pp.1090-1106. ISSN: 1747-6933.

[16] Karapetyants A., Samko S. Spaces BMOp(.)(D) of a variable exponent p(z). Georgian Math. J. 2010; 17:529-542.

[17] Karapetyants A., Samko S. Mixed norm Bergman - Morrey type spaces on the unit disc. Math. Notes, 100(1), pp.38-48 (2016). ISSN: 0001-4346.

De Rham Cohomology

Prof. Nicola Mazzari[1]

[1] University of Bordeaux I
Department of Maths
Email:Nicola.Mazzari@math.u-bordeaux1.fr

Timetable: 12 hrs. First lecture on February 21st, 2017, 13:30 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basis Commutive algebra. Basic Algebraic Topology. Basic Algebraic geometry and differential geometry.

Examination and grading: the exam will be oral and tailored on the basis of the student's attitude.

SSD: Mat02/03/05/07

Aim:
De Rham cohomology is at the basis of the duality between cycles and differentials. We will show the ubiquity of the de Rham cohomology in the arithmetic algebraic geometry setting. We wil show how to compute the de Rham complex ina complete algebraic way if we start with an algebraic geometry over the complex numbers. For this we will need to introduce some GAGA comparison theorems and some techniques of homological algebra.

Course contents:
De Rham cohomology is at the basis of the duality between cycles and differentials. We will show the ubiquity of the de Rham cohomology in the arithmetic algebraic geometry setting. We wil show how to compute the de Rham complex ina complete algebraic way if we start with an algebraic geometry over the complex numbers. For this we will need to introduce some GAGA comparison theorems and some techniques of homological algebra.

The Kolmogorov-Arnold-Moser theorem and its applications to the N-body problem

Prof. Gabriella Pinzari[1]

[1] *Dipartimento di Matematica, Universitá di Padova*
Email: pinzari@math.unipd.it

Timetable: 16 hours, First lecture on June 14, 2017, 11:00 (dates already fixed, see the calendar) Torre Archimede, Room 2BC/30.

Course requirements: principles of Hamiltonian Mechanics and Mathematical Analysis

Examination and grading: the exam will consist of a 45' lecture by the Student about one of the themes of the course, followed by questions

SSD: MAT/07

Aim: the purpose of the Course will be to give a view on KAM theory and the questions concerning with its applications to Celestial Mechanics. In particular, we shall review the story of the proof of a famous statement by V.I.Arnold in the 60s about the stability of planetary problem.

Course contents:
After recalling the notion *integrable systems and Liouville-Arnold integrable systems* [1], the course will be focused on the theory by Kolmogorov-Arnold-Moser (KAM) concerning the conservation of quasi-periodic motions to slightly perturbed systems.
A detailed proof of the theorem will be given, following the original ideas of A.N. Kolmogorov [11, 4]. Afterwards, we shall expose Arnold's proof, deserving particular attention to the case of "properly-degenerate" Hamiltonians arising from problems of Celestial Mechanics [2, 3].
Finally, we shall turn to the application to the Hamiltonian governing the motions of n small masses ("planets") interacting with a larger one ("sun").
The main steps of the long proof of a "theorem" stated in 1963 by V.I. Arnold on the stability of planetary motions, using kam theory, will be described [2, 9, 12, 14, 6].

References

[1] V. I. Arnold. A theorem of Liouville concerning integrable problems of dynamics. *Sibirsk. Mat.* $\hat{Z}$., **4**:471-474, 1963.

[2] V.I. Arnold. Small denominators and problems of stability of motion in classical and celestial mechanics. *Russian Math. Surveys*, **18**(6):85-191, 1963.

[3] L. Chierchia and G. Pinzari. Properlydegenerate KAM theory following V.I. Arnold. *Discrete Contin. Dyn. Syst. Ser. S*, **3**(4):545-578, 2010.

[4] L. Chierchia. A. N. Kolmogorov's 1954 paper on nearly-integrable Hamiltonian systems. A comment on: "On conservation of conditionally periodic motions for a small change in Hamilton's function" [Dokl. Akad. Nauk SSSR (N.S.) **98** (1954), 527-530; mr0068687]. *Regul. Chaotic Dyn.*, 13(2):130-139, 2008.

[5] L. Chierchia and G. Pinzari. Deprit's reduction of the nodes revisited. *Celest. Mech. Dyn. Astr.*, **109**(3):285-301, 2011.

[6] L. Chierchia and G. Pinzari. The planetary N-body problem: symplectic foliation, reductions and invariant tori. *Invent. Math.*, **186**(1):1-77, 2011.

[7] L. Chierchia and G. Pinzari. Metric stability of the planetary n-body problem. Proceedings of the International Congress of Mathematicians, 2014.

[8] A. Deprit. Elimination of the nodes in problems of n bodies. *Celestial Mech.*, **30**(2):181-195, 1983.

[9] J. Féjoz. Démonstration du "théorème d'Arnold" sur la stabilité du système planétaire (d'après Herman). *Ergodic Theory Dynam. Systems*, **24**(5):1521-1582, 2004.

[10] H. Hofer, E. Zehnder. Symplectic Invariants and Hamiltonian Dynamics. Birkhäuser Verlag, Basel, 1994.

[11] A. N. Kolmogorov. On the conservation of conditionally periodic motions under small perturbation of the Hamiltonian, *Dokl. Akad. Nauk. SSR*, **98** (1954), 527-530.

[12] J. Laskar and P. Robutel. Stability of the planetary three-body problem. I. Expansion of the planetary Hamiltonian. *Celestial Mech. Dynam. Astronom.*, **62**(3):193-217, 1995.

[13] G. Pinzari. Canonical coordinates for the planetary problem. *Acta Appl. Math.*, **137**:205-232, 2015.

[14] P. Robutel. Stability of the planetary three-body problem. II. KAM theory and existence of quasiperiodic motions. *Celestial Mech. Dynam. Astronom.*, **62**(3):219-261, 1995.

A soft introduction to algebraic entropy

Prof. Luigi Salce[1]

[1] *University of PADOVA*
Department of Maths
Email: salce@math.unipd.it

Timetable: 12 hrs. First lecture on February 21, 2017, 10:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.

Course requirements: Linear Algebra, Basic Algebra.

Examination and grading: Seminar on a subject assigned by the Instructor.

SSD: Mat/02; Mat/03

Aim: he course is an introduction to the theory of algebraic entropy of endomorphisms of algebraic structures in the basic setting of vector spaces over a field K. The two main results on this topic are presented: the Addition Theorem and the characterization of the algebraic entropy as the unique additive invariant extending the dimension invariant via the Bernoulli functor from the category of K-vector spaces to the category of K[X]-modules. Extensions to endomorphisms of Abelian goups and modules are outlined.

Course contents:

1. Preliminaries on vector spaces, modules over PIDs and the Fekete Lemma.
2. The category of flows of a linear transformation. The Bernoulli shift and the Bernoulli functor.
3. Definition, existence and properties of the algebraic entropy.
4. Algebraic entropy as rank of K[X]-modules. Addition and Uniqueness Theorems.
5. Adjoint algebraic entropy.
6. Algebraic entropies on Abelian groups and modules.

Weak KAM and Aubry-Mather Theory

Prof. Alfonso Sorrentino[1], Dott.ssa Olga Bernardi[2]

[1] *Universitá degli Studi di Roma "'Tor Vergata"'*
Dipartimento di Matematica
Email: sorrentino@math.uniroma2.it

[2] *Universitá degli Studi di Padova*
Dipartimento di Matematica
Email: obern@math.unipd.it

Timetable: 10 hrs (part 1: 6 hrs, by prof.Sorrentino + part 2: 4 hrs, by dott.ssa Bernardi).
(Part one) First lecture on March 15th, 2017, 14:30 (dates already fixed, see the calendar), Torre Archimede, Meeting Room VII floor.
(Part two) Dates to be confirmed, Torre Archimede.

Course requirements:

Examination and grading:

SSD: MAT/07

Aim:
[Part 1: Lectures 1-3, Prof. Alfonso Sorrentino] In these lecture we discuss John Mathers variational approach to the study of convex and superlinear Hamiltonian systems, what is generally called Aubry-Mather theory. Starting from the observation that invariant Lagrangian graphs can be characterised in terms of their action-minimizing properties, we shall describe how analogue features can be traced in a more general setting, namely the so-called Tonelli Hamiltonian systems. This approach brings to light a plethora of compact invariant subsets for the system, which, under many points of view, can be seen as a generalisation of invariant Lagrangian graphs, despite not being in general either submanifolds or regular. Besides being very significant from a dynamical systems point of view, these objects also appear and play an important role in many other different contexts: such as analysis, geometry, mathematical physics, etc. In particular, we shall describe the PDE counterpart of this variational approach, based on the study of weak solutions and subsolutions of the Hamilton-Jacobi equation: the so called weak KAM theory, developed by Albert Fathi. Time permitting, we shall also see how similar results can be extended to some non-conservative setting: the case of conformally symplectic systems.

[Part 1: Lectures 4-5, Dott.ssa Olga Bernardi] In the last two lectures we introduce some recent developments of Aubry-Mather theory. Starting from the min-max formula for the so-called effective Hamiltonian, we introduce and illustrate the dynamical content of Evans' approximate variational principle for Weak KAM theory. Moreover, we discuss the "boundary rigidity phenomenon" for convex hypersurfaces and some links with Lyapunov functions for the corresponding dynamics.

Course contents:

- Tonelli Lagrangian and Hamiltonian on compact manifolds.
- From KAM theory to Aubry-Mather theory: action-minimizing properties of invariant Lagrangian graphs.
- Mather theory: Action-minimizing invariant measures, Mather sets and minimal average actions.

- Weak KAM theory: Hamilton-Jacobi equation, weak (sub)solutions, action-minimizing curves, Aubry sets and Mañ sets.
- Aubry-Mather theory for conformally symplectic systems.
- Evans' approximate variational principle for Weak KAM theory.
- Recurrence phenomena and rigidity of Lagrangian submanifolds for convex hypersurfaces.

References:

1. C. Evans: *Some new PDE methods for weak KAM theory*, Calculus of Variations and PDE, 17 (2003), 159-177.
2. A. Fathi: *Weak KAM theory in Lagrangian dynamics* (Unpublished notes)
3. G.P. Paternain, L. Polterovich, and K.F. Siburg: *Boundary rigidity for Lagrangian submanifolds, non-removable intersections, and Aubry-Mather theory*, Mosk. Math. J. 3 (2003), 593-619.
4. A. Sorrentino *Action-Minimizing Methods in Hamiltonian Dynamics. An Introduction to Aubry-Mather Theory*. Mathematical Notes Series Vol. 50 (Princeton University Press), 2015.

Courses in collaboration with the Doctoral School on “Information Engineering”

Applied Linear Algebra

Prof. F. De Terán[1], Prof. Michael Karow[2]

[1] Universidad Carlos III de Madrid, Spain
Email: fteran@math.uc3m.es

[2] Technische Universität Berlin, Germany
Email: karow@math.tu-berlin.de

Timetable: 16 hours. First lecture on March, Monday 21st 4:30 am. Second Lecture on March, Wednedsday 23rd, 2.30 pm. All other 6 lectures on Wednesday (2.30 pm) and Friday (10.30 am) starting from March, Wednesday 30th 2016 (Dept. of Information Engineering, Via Gradenigo 6/a, Padova)

Course requirements: A good working knowledge of basic notions of linear algebra as for example in [1]. Some proficiency in MATLAB.

Examination and grading: Grading is based on homeworks or a written examination or both.

Aim: We study concepts and techniques of linear algebra that are important for applications with special emphasis on the topics *low rank approximation* and *matrix equations and inequalities*. A wide range of exercises and problems will be an essential part of the course and constitute homework required to the student.

Course contents:

- Review of some basic concepts of linear algebra and matrix theory
- The singular value decomposition and applications
- Krylov subspaces
- Matrix equations and matrix inequalities
- Sylvester and Lyapunov equations, Riccati equation, linear matrix inequalities (LMIs)

References:

[1] Gilbert Strang's linear algebra lectures, from M.I.T. on You Tube

[2] Notes from the instructors

Bayesian Machine Learning

Prof. Giorgio Maria Di Nunzio[1]

[1] *Dept. of Information Engineering, University of Padova*
Email: dinunzio@dei.unipd.it

Timetable: Course of 20 hours. Tentative schedule: Class meets every Thursday from 14:30 to 16:30 and Friday from 11:30 to 13:30. First lecture on Thursday, 12th January, 2017. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: Basics of Probability Theory. Basics of R Programming.

Examination and grading: Homework assignments and final project.

Aim: The course will introduce fundamental topics in Bayesian reasoning and how they apply to machine learning problems. In this course, we will present pros and cons of Bayesian approaches and we will develop a graphical tool to analyse the assumptions of these approaches in practical problems.

Course contents:

- *Introduction of classical machine learning problems.*
  - Mathematical framework
  - Supervised and unsupervised learning
- *Bayesian decision theory*:
  - Two-category classification
  - Minimum-error-rate classification
  - Bayes risk
  - Decision surfaces
- *Estimation*:
  - Maximum Likelihood Estimation
  - Maximum A Posteriori
  - Bayesian approach
- *Graphical models*:
  - Bayesian networks
  - Two-dimensional probabilistic model
- *Evaluation* :
  - Measures of accuracy
  - Statistical signifiance testing

References:

[1] J. Kruschke, Doing Bayesian Data Analysis: A Tutorial Introduction With R and Bugs, Academic Press 2010

[2] Christopher M. Bishop, Pattern Recognition and Machine Learning (In- formation Science and Statistics), Springer 2007

[3] Richard O. Duda, Peter E. Hart, David G. Stork, Pattern Classification (2nd Edition), Wiley-Interscience, 2000

[4] Yaser S. Abu-Mostafa, Malik Magdon-Ismail, Hsuan-Tien Lin, Learning from Data, AML-Book, 2012 (supporting material available at http://amlbook.com/support.html)

[5] David J. C. MacKay, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003 (freely available and supporting material at http://www.inference.phy.cam.ac.uk/mackay/

[6] David Barber, Bayesian Reasoning and Machine Learning, Cambridge University Press, 2012 (freely available at http://web4.cs.ucl.ac.uk/staff/D.Barber/pmwiki/pmwiki.php?n=

[7] Kevin P. Murphy, Machine Learning: A Probabilistic Perspective, MIT Press, 2012 (supporting material http://www.cs.ubc.ca/ murphyk/MLbook/)

Statistical methods

Prof. Lorenzo Finesso[1]

[1] *Istituto di Elettronica e di Ingegneria dell'Informazione e delle Telecomunicazioni, IEIIT-CNR, Padova*
Email: lorenzo.finesso@unipd.it

Timetable: Course of 24 hrs . Class meets every Monday and Wednesday from 10:30 to 12:30. First lecture on April 19th, 2017. Room DEI/G (3-rd floor, Dept. of Information Engineering, via Gradenigo Building).

Course requirements: familiarity with basics linear algebra.

Examination and grading: homework and take-home exam.

Aim: The course will present a small selection of statistical techniques which are widespread in applications. The unifying power of the information theoretic point of view will be stressed.

Course contents:

- *Background material.* The noiseless source coding theorem will be quickly reviewed in order to introduce the basic notions of entropy and I-divergence
- *Divergence minimization problems.* Three I-divergence minimization problems will be posed and, via examples, they will be connected with basic methods of statistical inference: ML (maximum likelihood), ME (maximum entropy), and EM (expectation-maximization).
- *Multivariate analysis methods.* The three standard multivariate methods, PCA (Principal component analysis), Factor Analysis, and CCA (Canonical Correlations analysis) will be reviewed and their connection with divergence minimization discussed. Applications of PCA to least squares (PCR principal component regression, PLS Partial least squares). Approximate matrix factorization and PCA, with a brief detour on the approximate Nonnegative Matrix Factorization (NMF) problem. The necessary linear algebra will be reviewed.
- *EM methods.* The Expectation-Maximization method will be introduced as an algorithm for the computation of the Maximum Likelihood (ML) estimator with partial observations (incomplete data) and interpreted as an alternating divergence minimization algorithm (à la Csiszár Tusnády).
- *Applications to stochastic processes.* Introduction to HMM (Hidden Markov Models). Maximum likelihood estimation for HMM via the EM method. If time allows: derivation of Burg spectral estimation method as solution of a Maximum Entropy problem.

References: A set of lecture notes and a complete list of references will be posted on the web site of the course.

Optimization and Optimal Control

Prof. John Hauser[1]

[1] *University of Colorado Boulder*
ducati.motogp@gmail.com

Timetable: Course of 20 hours. Class meets every Tuesday and Thursday from 10.30 to 12.30. First lecture on April, 11th, 2017. Room DEI/G, 3rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: familiarity with basic linear algebra.

Examination and grading: homework and final project.

Aim: In this course, we will study the use of a nonlinear projection operator in the development of a novel function space approach for the optimization of trajectory functionals. Given a bounded state-control trajectory of a nonlinear system, one may make use of a simple (e.g., linear time-varying) trajectory tracking control law to explore the set of nearby bounded state-control trajectories. Such a trajectory tracking control system defines a nonlinear projection operator that maps a set of bounded curves onto a set of nearby bounded trajectories.

Course contents:
We will use the projection operator approach to develop a Newton descent method for the optimization of dynamically constrained functionals. By projecting a neighboring set of state-control curves onto the trajectory manifold and then evaluating the cost functional, the constraint imposed by the nonlinear system dynamics is subsumed into an unconstrained trajectory functional. Attacking this equivalent optimization problem in an essentially unconstrained manner, we will discover an algorithm defined in function space that produces a descending sequence in the Banach manifold of bounded trajectories. The specific computations for this algorithm will be implemented by solving ordinary differential equations. Of special interest is the trajectory representation theorem: trajectories near a given trajectory can be represented uniquely as the projection of the sum of that trajectory and a tangent trajectory, providing a local chart for the trajectory manifold. The composition of the cost functional with this mapping is thereby a mapping from the Banach space of tangent trajectories into the real numbers and it is this local mapping that may or may not possess (local) convexity properties. When the second Frechet derivative of this mapping is positive definite (in an appropriate sense), the mapping is locally convex which is useful for many applications including the existence of a Newton descent direction, second order sufficient condition (SSC) for optimality, quadratic convergence, and continuous dependence of optimal trajectories on initial conditions. We will make use of the PRojection Operator based Newton method for Trajectory Optimization (PRONTO) to do some numerical ”trajectory exploration” on some interesting nonlinear systems, including possible student selected systems. Throughout the course, various concepts will be illustrated with examples and followed by homework assignments designed to enhance understanding.

References: Lecture notes and references will be posted on the web site of the course.

Computational Inverse Problems

Prof. Fabio Marcuzzi[1]

[1] University of Padova
Department of Mathematics
Email: marcuzzi@math.unipd.it

Timetable: 20 hrs (2 two-hours lectures per week): Classes on Monday and Wednesday, 10:30 - 12:30. First lecture on Monday February 27th, 2017. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: basic notions of linear algebra and, possibly, numerical linear algebra. The examples and homework will be in Python (the transition from Matlab to Python is effortless).

Examination and grading: Homework assignments and final test.

SSD: MAT/08

Aim: We study numerical methods that are of fundamental importance in computational inverse problems. Real application examples will be given for distributed parameter systems. Computer implementation performance issues will be considered also.

Course contents:

- definition of inverse problems, basic examples and numerical difficulties.
- numerical methods for QR and SVD and their application to the squareroot implementation in PCA, least-squares, model reduction and Kalman filtering; recursive least-squares;
- regularization methods;
- numerical algorithms for nonlinear parameter estimation: Gauss-Newton, Levenberg-Marquardt;
- examples with distributed parameter systems;
- HPC implementations.

References:

[1] F.Marcuzzi "Analisi dei dati mediante modelli matematici",
`http://www.math.unipd.it/˜ marcuzzi/MNAD.html`

Applied Functional Analysis and Machine Learning

Prof. Gianluigi Pillonetto[1]

[1] *Dept. of Information Engineering, University of Padova*
Email: giapi@dei.unipd.it

Timetable: 28 hrs (2 two-hours lectures per week): Classes on Tuesday and Thursday, 10:30 - 12:30. First lecture on Thursday November 24th, 2016. Sala Riunioni 318 DEI/G 3-rd oor, via Gradenigo 6).

Course requirements:

1. The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence.
2. The arithmetic of complex numbers and the basic properties of the complex exponential function.
3. Some elementary set theory.
4. A bit of linear algebra.

All the necessary material can be found in W. Rudin's book Principles of Mathematical Analysis (3rd ed., McGraw-Hill, 1976). A summary of the relevant facts will be given in the first lecture.

Examination and grading: Homework assignments and final test.
Students wishing to take the exam for this course, are kindly requested to contact the Coordinator of the PhD program in Mathematical Sciences,

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

Course contents:

1. *Review of some notions on metric spaces and Lebesgue integration*: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.
2. *Banach and Hilbert spaces*: Normed spaces and Banach spaces. Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces.
3. *Compact linear operators on normed spaces and their spectrum*: Spectral properties of bounded linear operators. Compact linear operators on normed spaces. Spectral properties of compact linear operators. Spectral properties of bounded self-adjoint operators, positive operators, operators defined by a kernel. Mercer Kernels and Mercer's theorem.
4. *Reproducing kernel Hilbert spaces, inverse problems and regularization theory*: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Primal and dual formulation of loss functions. Regularization networks. Consis-

tency/generalization and relationship with Vapnik's theory and the concept of V_γ dimension. Support vector regression and classification.

References:

[1] W. Rudin. Real and Complex Analysis, McGraw Hill, 2006.

[2] E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons, 1978.

[3] G. Wahba. Spline models for observational data. SIAM, 1990.

[4] C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006.

[5] R.T. Rockafellar. Convex analysis. Princeton University Press, 1996.

Local Methods for Nonlinear Systems and Control

Prof. Rodolphe Sepulchre[1], Prof. Fulvio Forni[2]

[1] *Dept. of Information Engineering, University of Cambridge, UK*
Email: f.forni@eng.cam.ac.uk

[2] *Dept. of Information Engineering, University of Cambridge, UK*
Email: r.sepulchre@eng.cam.ac.uk

Timetable: March 27-March 31, 2017. Room DEI/G, 3-rd floor, Dept. of Information Engineering, via Gradenigo Building.
Site of the Course: http://www.dei.unipd.it/ zorzimat/EECIPD/Course.pdf

Abstract of the course
Linear modeling is both the success and the limitation of engineering: local methods are analytically and computationally efficient but a thousand linearized models do not necessary account for a global phenomenon. The course will present a theory of nonlocal phenomena that can be analyzed by means of local methods, with the aim of enlarging the modeling and design principles of nonlinear control science. The emphasis will be on the role of symmetry and feedback in localizing architectures.

Topics

- **Lecture 1: Local and global analysis.**
  The principle of linearization. Review of local and global analysis methods. Motivations for local methods in global analysis. Invariance and feedback as two fundamental guidelines for local methods.
- **Lecture 2: The line, the ray, and the circle**
  Nonlinear spaces that possess efficient linearizations. Elements of differential geometry and Lie group theory. Important homogeneous spaces encountered in engineering.
- **Lecture 3: Local calculus made non local.**
  Averaging, filtering, and interpolating in nonlinear spaces.
- **Lecture 4: Local calculus made non local.**
  Differential Stability Systems that forget initial conditions. Lyapunov stability, contraction analysis, incremental stability.
- **Lecture 5: Differential Positivity.**
  Systems that preserve an order. Perron Frobenius theory. Monotonicity. Conal orders.
- **Lecture 6: Differential analysis of open systems.**
  Stability and positivity for open systems, interconnections, dissipativity.
- **Lecture7. Excitability** Locallisation by feedback. Balance between positive and negative feedback. Balance between stability and positivity. Models of excitability. Interconnections.

Calendar

The calendar is not completely filled in. Updated June 19, 2017

ottobre 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
25	26	27	28	29	30	1
2	3	4	5 * Seminario Dottorato (Matematica Computazionale) - 16:00-17:30	6	7	8
9	10	11	12	13 * Vargiolu 14:30-16:30	14	15
16	17	18	19	20 * Vargiolu 15:30-17:30	21	22
23	24	25	26	27 * Vargiolu 15:30-17:30	28	29
30	31	1	2	3	4	5

novembre 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
30	31	1	2 * Vargiolu 14:30-16:30	3 * Vargiolu 15:30-17:30	4	5
6	7	8 * Tudisco 11:00-13:00	9	10 * Tudisco 11:00-13:00	11	12
13	14	15 * Tudisco 14:00-16:00	16 * Seminario Dottorato (Matematica) - 14:30-16:00 * Chiarellotto 11:00-13:00	17 * Tudisco 11:00-13:00 * Chiarellotto 09:00-11:00	18 * Bardi 11:00-13:00	19
20	21 * Conforti 14:00-16:00	22 * Tudisco 14:00-16:00 * Bardi 09:00-11:00	23 * Conforti 14:00-16:00 * Chiarellotto 11:00-13:00	24 * Chiarellotto 11:00-13:00 * Conforti 14:00-16:00 * Pillonetto 10:30-12:30	25 * Tudisco 11:00-13:00	26
27	28 * Conforti 14:00-16:00	29 * Conforti 14:00-16:00 * Bardi 11:00-13:00 * Pillonetto 10:30-12:30	30 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Chiarellotto 11:00-13:00	1	2	3
4	5	6	7	8	9	10

dicembre 2016

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
27	28	29	30	1 * Pillonetto 10:30-12:30 * Conforti 14:00-16:00 * Chiarellotto 11:00-13:00	2	3
4	5	6 * Pillonetto 10:30-12:30 * Chiarellotto 11:00-13:00 * Bardi 09:00-11:00	7 * Chiarellotto 09:00-11:00	8	9	10
11	12	13 * Pillonetto 10:30-12:30	14 * Seminario Dottorato (Matematica) - 14:30-16:00 * Chiarellotto 11:00-13:00	15 * Chiarellotto 11:00-13:00 * Pillonetto 10:30-12:30	16 * Bardi 11:00-13:00	17
18	19	20 * Pillonetto 10:30-12:30	21	22 * Pillonetto 10:30-12:30	23	24
25	26	27	28	29	30	31
1	2	3	4	5	6	7

gennaio 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
1	2	3	4	5	6	7
8	9	10 * Pillonetto 10:30-12:30 * Kroo 10:30-12:30	11 * Kroo 11:30-13:30	12 * Di Nunzio 14:30-16:30 * Pillonetto 10:30-12:30 * Kroo 11:30-13:30	13 * Bardi 11:00-13:00 * Kroo 09:00-11:00 * Di Nunzio 11:30-13:30	14
15	16	17 * Kroo 10:30-12:30 * Pillonetto 10:30-12:30	18 * Kroo 11:30-13:30 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00	19 * Pillonetto 10:30-12:30 * Kroo 11:30-13:30 * Di Nunzio 14:30-16:30	20 * Bardi 11:00-13:00 * Kroo 09:00-11:00 * Di Nunzio 11:30-13:30	21
22	23	24 * Pillonetto 10:30-12:30	25	26 * Di Nunzio 14:30-16:30 * Pillonetto 10:30-12:30	27 * Bardi 11:00-13:00 * Di Nunzio 11:30-13:30	28
29	30	31	1	2	3	4
5	6	7	8	9	10	11

febbraio 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
29	30	31	1 * Seminario Dottorato (Matematica) - 14:30-16:00	2 * Di Nunzio 14:30-16:30	3 * Di Nunzio 11:30-13:30 * Bardi 11:00-13:00	4
5	6	7	8	9 * Di Nunzio 14:30-16:30	10 * Di Nunzio 11:30-13:30	11
12	13 * Achdou 11:00-13:00	14	15 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00	16 * Achdou 11:00-13:00	17	18
19	20 * Achdou 11:00-13:00	21 * Mazzari 13:30-16:30 * Salce 10:00-12:00	22 * Achdou 11:00-13:00	23	24	25
26	27 * Karapetyants 10:00-12:00 * Marcuzzi 10:30-12:30	28 * Salce 10:30-12:30 * Karapetyants 10:00-13:00 * Mazzari 13:30-16:30	1	2	3	4
5	6	7	8	9	10	11

marzo 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
26	27	28	1 * Seminario Dottorato (Matematica) - 14:30-16:00 * Marcuzzi 10:30-12:30	2 * Karapetyants 10:00-13:00	3 * Karapetyants 10:00-12:00	4
5	6 * Marcuzzi 10:30-12:30	7 * Mazzari 13:30-16:30 * Salce 10:30-12:30	8 * Marcuzzi 10:30-12:30	9	10	11
12	13 * Marcuzzi 10:30-12:30	14 * Sorrentino 15:00-17:00 * Mazzari 13:30-16:30 * Salce 10:30-12:30	15 * Sorrentino 15:00-17:00 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00 * Marcuzzi 10:30-12:30	16 * Sorrentino 15:00-17:00	17	18
19	20 * Marcuzzi 10:30-12:30	21 * Teran-Karow 16:30-18:30 * Salce 10:30-12:30	22 * Marcuzzi 10:30-12:30	23 * Teran-Karow 14:30-16:30	24	25
26	27 * Marcuzzi 10:30-12:30	28 * Salce 10:30-12:30	29 * Seminario Dottorato (Matematica) - 14:30-16:00 * Teran-Karow 14:30-16:30 * Marcuzzi 10:30-12:30	30	31 * Teran-Karow 10:30-12:30	1
2	3	4	5	6	7	8

aprile 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
26	27	28	29	30	31	1
2	3 * Carnovale 09:30-11:15 * Kardaras 16:30-18:00	4 * Kardaras 16:30-18:00	5 * Teran-Karow 14:30-16:30 * Kardaras 16:30-18:00	6 * Carnovale 11:30-13:15 * Kardaras 16:30-18:00	7 * Teran-Karow 10:30-12:30 * Kardaras 16:0-17:30	8
9	10 * Carnovale 09:30-11:15	11 * Hauser 10:30-12:30	12 * Teran-Karow 14:30-16:30	13 * Hauser 10:30-12:30 * Carnovale 11:30-13:15	14 * Teran-Karow 10:30-12:30	15
16	17	18 * Hauser 10:30-12:30	19 * Finesso 10:30-12:30	20 * Carnovale 11:30-13:15 * Hauser 10:30-12:30	21	22
23	24 * Finesso 10:30-12:30	25 * Hauser 10:30-12:30	26 * Finesso 10:30-12:30	27 * Carnovale 11:30-13:15 * Hauser 10:30-12:30	28	29
30	1	2	3	4	5	6

maggio 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
30	1	2 * Hauser 10:30-12:30	3 * Finesso 10:30-12:30 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00	4 * Carnovale 11:30-13:15 * Hauser 10:30-12:30	5	6
7	8 * Finesso 10:30-12:30 * Carnovale 09:30-11:15	9 * Maas 13:30-17:30 * Hauser 10:30-12:30	10 * Finesso 10:30-12:30	11 * Hauser 10:30-12:30 * Carnovale 11:30-13:15 * Maas 13:30-17:30	12	13
14	15 * Finesso 10:30-12:30 * Carnovale 09:30-11:15	16	17 * Seminario Dottorato (Matematica) - 14:30-16:00 * Finesso 10:30-12:30	18 * Carnovale 11:30-13:15	19	20
21	22 * Carnovale 11:30-13:15 * Finesso 10:30-12:30	23	24 * Finesso 10:30-12:30	25	26	27
28	29 * Finesso 10:30-12:30	30	31 * Finesso 10:30-12:30 * Seminario Dottorato (Matematica Computazionale) - 14:30-16:00	1	2	3
4	5	6	7	8	9	10

giugno 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
28	29	30	31	1	2	3
4	5 * Gaudilliére 11:00-13:00	6 * Gaudilliére 11:00-13:00	7 * Gaudilliére 14:30-16:30	8 * Gaudilliére 14:30-16:30	9 * Gaudilliére 11:00-13:00	10
11	12	13	14 * Seminario Dottorato (Matematica) - 14:30-16:00 * Pinzari 11:00-13:00	15	16 * Pinzari 11:00-13:00	17
18	**19**	20	21	22 * Gaudilliére 09:00-11:00	23 * Gaudilliére 11:00-13:00 * Pinzari 10:30-14:30	24
25	26 * Gaudilliére 11:00-13:00	27 * Gaudilliére 14:30-16:30 * Pinzari 11:00-13:00	28 * Gaudilliére 14:30-16:30	29 * Gaudilliére 14:30-16:30 * Pinzari 11:00-13:00	30 * Gaudilliére 11:00-13:00	1
2	3	4	5	6	7	8

luglio 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
25	26	27	28	29	30	1
2	3	4 * Pinzari 11:00-13:00	5	6 * Pinzari 11:00-13:00	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31	1	2	3	4	5

agosto 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31	1	2
3	4	5	6	7	8	9

settembre 2017

domenica	lunedì	martedì	mercoledì	giovedì	venerdì	sabato
27	28	29	30	31	1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30 * Fine anno accademico
1	2	3	4	5	6	7