Optimization and Optimal Control

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Timetable: Course of 20 hours. Class meets every Tuesday and Thursday from 10.30 to 12.30. First lecture on April, 11th, 2017. Room DEI/G, 3rd floor, Dept. of Information Engineering, via Gradenigo Building.

Course requirements: familiarity with basic linear algebra.

Examination and grading: homework and final project.

Aim: In this course, we will study the use of a nonlinear projection operator in the development of a novel function space approach for the optimization of trajectory functionals. Given a bounded state-control trajectory of a nonlinear system, one may make use of a simple (e.g., linear time-varying) trajectory tracking control law to explore the set of nearby bounded state-control trajectories. Such a trajectory tracking control system defines a nonlinear projection operator that maps a set of bounded curves onto a set of nearby bounded trajectories.

Course contents:
We will use the projection operator approach to develop a Newton descent method for the optimization of dynamically constrained functionals. By projecting a neighboring set of state-control curves onto the trajectory manifold and then evaluating the cost functional, the constraint imposed by the nonlinear system dynamics is subsumed into an unconstrained trajectory functional. Attacking this equivalent optimization problem in an essentially unconstrained manner, we will discover an algorithm defined in function space that produces a descending sequence in the Banach manifold of bounded trajectories. The specific computations for this algorithm will be implemented by solving ordinary differential equations. Of special interest is the trajectory representation theorem: trajectories near a given trajectory can be represented uniquely as the projection of the sum of that trajectory and a tangent trajectory, providing a local chart for the trajectory manifold. The composition of the cost functional with this mapping is thereby a mapping from the Banach space of tangent trajectories into the real numbers and it is this local mapping that may or may not possess (local) convexity properties. When the second Frechet derivative of this mapping is positive definite (in an appropriate sense), the mapping is locally convex which is useful for many applications including the existence of a Newton descent direction, second order sufficient condition (SSC) for optimality, quadratic convergence, and continuous dependence of optimal trajectories on initial conditions. We will make use of the PRojection Operator based Newton method for Trajectory Optimization (PRONTO) to do some numerical ”trajectory exploration” on some interesting nonlinear systems, including possible student selected systems. Throughout the course, various concepts will be illustrated with examples and followed by homework assignments designed to enhance understanding.

References: Lecture notes and references will be posted on the web site of the course.