Bergman spaces of analytic functions: from classical results to the new settings and ideas involving nonstandard behavior of function and symbols of operators.

Prof. Alexey Karapetyants[1]

[1] *Southern Federal University and Don State Technical University, Russia*
Email: karapetyants@gmail.com

Timetable: 10 hours. First lecture on February 27, 2017, 10:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30

Course requirements: Knowledge of basic university facts on functions of real and complex variables, basic properties of Banach and Hilbert spaces, basic properties of linear operators on Banach and Hilbert spaces. Everything on the level of understanding basic university courses which include these subjects.

Examination and grading: During the course I will ask some questions for homework activity. Those who will complete these homework questions will get score up to 50% of the total. The final test (quiz) will be given to the students, and on the base of this test a student may have also up to 50% of the total score. Those students who will consider their score lower than they've expected will be provided with an opportunity to defend their answers with no restrictions on time after the last lecture.

SSD: MAT/05

Aim: The aim of the course is twofold. First, to provide the students with classical objects and classical (textbook) instruments on study of Bergman spaces and operators in these spaces. Second, to provide with a knowledge of further development of the study of Bergman type spaces and operators in nonstandard situations, where the symbols of operators or the functions from these spaces admit so-called nonstandard behavior in a neighborhood of the boundary.

Description of the Course:
Agenda (a short review of planned research)
This short lecture course is aimed to introduce the students with basic knowledge of classical Bergman spaces and Toeplitz operators, and to provide the motivation and recent results for developing of the theory of analytic Bergman type spaces of functions with nonstandard (non-classical) boundary behavior. The course is based on classical textbooks for Bergman spaces and operator theory, papers by the author in collaboration with N.Vasilevsky and S.Grudsky on Toeplitz operators with badly behaved special symbols, and papers by the author with S.Samko on some new mixed norm Bergman type spaces of functions with nonstandard growth near the boundary.

Work plan
This work plan is for either five lectures each consisting of two hours or for ten one-hour lectures.

1. Basic facts on classical Bergman spaces and Toeplitz type operators (4 hours)
   This part is aimed to provide background information for Bergman type spaces and classical Toeplitz operators. It includes the discussion of classical Bergman spaces and Toeplitz type operators with nice symbols, the Bergman kernel function, point evaluation functional, orthonormal bases, boundedness of Bergman projection, the Berezin transform of the Toeplitz operator and its connection with properties of Toeplitz operator. These basic results will be followed with an outline of some advanced results on special Toeplitz operators with unbounded and badly behaved symbols, which also serves as a motivation for further development of the theory in the direction of mixed norm and nonstandard growth space settings.
2. The definition and basic properties of new Bergman type mixed norm spaces: general approach (2 hours)
   This two hours lecture is aimed to providing a new general approach to the definition of mixed norm Bergman space on the unit disc in a general settings. We discuss the problem of boundedness of Bergman projection in this general setting, the meaning of distributional Fourier coefficients used in the definition of the spaces, completeness of these new spaces, and other basic facts.
3. Concrete new Bergman type mixed norm spaces (4 hours)
   We start with the definitions and basic properties of particular spaces of functions whose norms are used in the definition of the new Bergman type mixed norm spaces: variable Lebesgue spaces, Orlich and Morrey type spaces. Further we discuss each particular case (Bergman-variable Lebesgue, Bergman-Orlich, Bergman-Morrey) separately proving the boundedness of the Bergman projection, characterizing the function in these spaces and discussing the boundary behavior of functions in such spaces. At the conclusion we propose some open questions and provide with a short resume of future plans for research in this new direction of study of spaces and operators.

References for parts 1 and 2:

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[4] Zhu K. Operator theory in function spaces. Mathematical Surveys and Monographs, Vol. 138; 2007.

[5] Vasilevski N.L. Commutative Algebras of Toeplitz Operators on the Bergman Space. Vol.185: Operator Theory: Advances and Applications; 2008.

[6] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Toeplitz operators on the unit ball in $\mathbb{C}^n$ with radial symbols. J. Operator Theory. 2003; 49:325-346.

[7] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Dynamics of properties of Toeplitz operators with radial symbols. Integral Equations Operator Theory. 2004; 50:217-253.

[8] Grudsky S.M., Karapetyants A.N., Vasilevski N.L. Dynamics of properties of Toeplitz operators in the weighted Bergman spaces. Siberian Electronic Mathematical Reports. 2006; 3:362-383.

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References for part 3:

[11] Diening L., Harjulehto P., Hasto P., Ruzicka M. Lebesgue and Sobolev spaces with variable exponents. Vol.2017. Springer-Verlag, Heidelberg: Lecture Notes in Mathematics; 2011.

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[14] Kokilashvili V., Meskhi A., Rafeiro H., Samko S. Integral Operators in Non-Standard Function Spaces. Vol. II: Variable Exponent Holder, Morrey–Campanato and Grand Spaces. Birkhauser Basel: Operator Theory: Advances and Applications; 2016.

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