Option Pricing: from Monte Carlo to Quantization

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Timetable: 24 hours, Spring 2017, at Politecnico di Milano.

Course requirements: The course assumes the knowledge of basic probability.

Examination and grading: Code development on a specific project.

SSD: SECS-S/06

Aim:
Monte Carlo methods are extensively used in finance to value and analyze complex instruments, portfolios and investments by simulating the various sources of uncertainty affecting their value. The advantage of Monte Carlo methods over other techniques increases as the dimensions (sources of uncertainty) of the problem increase. However, it is well known that the disadvantage of Monte Carlo methods is the slow convergence, and thus the high computational cost of the algorithms. Quantization is a way to approximate a random vector or a stochastic process using a nearest neighbour projection on a finite codebook. The birth of quantization dates back to the 1950s, when in the Bell laboratories ad hoc signal discretization procedures were developed for signal transmission. In the last years, Quantization has been deeply considered in numerical probability, especially for solving problems arising in mathematical finance, presenting itself as a de facto alternative to Monte Carlo methods. The purpose of the course is to highlight the characteristics of the two methodologies and to deeply analyze (and implement) their applications in financial context, mainly option pricing.

Course contents:

Pre-course (if necessary/required): Matlab introduction

First part: Monte Carlo (MC) Methods (Daniele Marazzina)

- Introduction to financial derivatives and to financial stochastic modelling
- MC methods and variance reduction (control variates and antithetic sampling)
- MC codes: exact schemes, Euler and Milstein schemse, QE scheme for Heston
- MC for jump processes

Second part: Quantization (Giorgia Callegaro)

- Introduction to quantization
- Quantization of a random variable
- Marginal Recursive quantization
- Applications of Marginal Recursive Quantization: barrier options and exotic option pricing

Third part: Implementationof quantization methods (Lucio Fiorin)

- 1D setting: Black Scholes, CEV, pricing of European options, barrier and American options
- 2D case: Heston, pricing of European and American options

References: Graf, Siegfried, and Harald Luschgy. Foundations of quantization for probability distributions. Springer, 2000.