The Kolmogorov-Arnold-Moser theorem and its applications to the N-body problem

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Timetable: 16 hours, First lecture on June 14, 2017, 11:00 (dates already fixed, see the calendar) Torre Archimede, Room 2BC/30.

Course requirements: principles of Hamiltonian Mechanics and Mathematical Analysis

Examination and grading: the exam will consist of a 45' lecture by the Student about one of the themes of the course, followed by questions

SSD: MAT/07

Aim: the purpose of the Course will be to give a view on KAM theory and the questions concerning with its applications to Celestial Mechanics. In particular, we shall review the story of the proof of a famous statement by V.I.Arnold in the 60s about the stability of planetary problem.

Course contents:
After recalling the notion *integrable systems and Liouville-Arnold integrable systems* [1], the course will be focused on the theory by Kolmogorov-Arnold-Moser (KAM) concerning the conservation of quasi-periodic motions to slightly perturbed systems.
A detailed proof of the theorem will be given, following the original ideas of A.N. Kolmogorov [11, 4]. Afterwards, we shall expose Arnold's proof, deserving particular attention to the case of "properly-degenerate" Hamiltonians arising from problems of Celestial Mechanics [2, 3].
Finally, we shall turn to the application to the Hamiltonian governing the motions of n small masses ("planets") interacting with a larger one ("sun").
The main steps of the long proof of a "theorem" stated in 1963 by V.I. Arnold on the stability of planetary motions, using kam theory, will be described [2, 9, 12, 14, 6].

References

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