Weak KAM and Aubry-Mather Theory

Prof. Alfonso Sorrentino[1], Dott.ssa Olga Bernardi[2]

[1]Universitá degli Studi di Roma "'Tor Vergata"'
Dipartimento di Matematica
Email: sorrentino@math.uniroma2.it

[2] Universitá degli Studi di Padova
Dipartimento di Matematica
Email: obern@math.unipd.it

Timetable: 10 hrs (part 1: 6 hrs, by prof.Sorrentino + part 2: 4 hrs, by dott.ssa Bernardi).
(Part one) First lecture on March 15th, 2017, 14:30 (dates already fixed, see the calendar), Torre Archimede, Meeting Room IV floor.
(Part two) Dates to be confirmed, Torre Archimede.

Course requirements:

Examination and grading:

SSD: MAT/07

Aim:
[Part 1: Lectures 1-3, Prof. Alfonso Sorrentino] In these lecture we discuss John Mathers variational approach to the study of convex and superlinear Hamiltonian systems, what is generally called Aubry-Mather theory. Starting from the observation that invariant Lagrangian graphs can be characterised in terms of their action-minimizing properties, we shall describe how analogue features can be traced in a more general setting, namely the so-called Tonelli Hamiltonian systems. This approach brings to light a plethora of compact invariant subsets for the system, which, under many points of view, can be seen as a generalisation of invariant Lagrangian graphs, despite not being in general either submanifolds or regular. Besides being very significant from a dynamical systems point of view, these objects also appear and play an important role in many other different contexts: such as analysis, geometry, mathematical physics, etc. In particular, we shall describe the PDE counterpart of this variational approach, based on the study of weak solutions and subsolutions of the Hamilton-Jacobi equation: the so called weak KAM theory, developed by Albert Fathi. Time permitting, we shall also see how similar results can be extended to some non-conservative setting: the case of conformally symplectic systems.

[Part 1: Lectures 4-5, Dott.ssa Olga Bernardi] In the last two lectures we introduce some recent developments of Aubry-Mather theory. Starting from the min-max formula for the so-called effective Hamiltonian, we introduce and illustrate the dynamical content of Evans' approximate variational principle for Weak KAM theory. Moreover, we discuss the "boundary rigidity phenomenon" for convex hypersurfaces and some links with Lyapunov functions for the corresponding dynamics.

Course contents:

- Tonelli Lagrangian and Hamiltonian on compact manifolds.
- From KAM theory to Aubry-Mather theory: action-minimizing properties of invariant Lagrangian graphs.
- Mather theory: Action-minimizing invariant measures, Mather sets and minimal average actions.

- Weak KAM theory: Hamilton-Jacobi equation, weak (sub)solutions, action-minimizing curves, Aubry sets and Mañ sets.
- Aubry-Mather theory for conformally symplectic systems.
- Evans' approximate variational principle for Weak KAM theory.
- Recurrence phenomena and rigidity of Lagrangian submanifolds for convex hypersurfaces.

References:

1. C. Evans: *Some new PDE methods for weak KAM theory*, Calculus of Variations and PDE, 17 (2003), 159-177.
2. A. Fathi: *Weak KAM theory in Lagrangian dynamics* (Unpublished notes)
3. G.P. Paternain, L. Polterovich, and K.F. Siburg: *Boundary rigidity for Lagrangian submanifolds, non-removable intersections, and Aubry-Mather theory*, Mosk. Math. J. 3 (2003), 593-619.
4. A. Sorrentino *Action-Minimizing Methods in Hamiltonian Dynamics. An Introduction to Aubry-Mather Theory*. Mathematical Notes Series Vol. 50 (Princeton University Press), 2015.